

# At the Interface of Geometry and Analysis

Department of Mathematics  
University of Auckland

19-21 February 2025

Lectures by Richard Melrose (MIT)

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**Title:** TBA

**Abstract:** TBA

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Public Lecture

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# Abstracts

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**Speaker:** Qiongling Li (Chern Institute of Mathematics, Nankai University)

**Title:** Index and total curvature of minimal surfaces in noncompact symmetric spaces and wild harmonic bundles

**Abstract:** We prove two main theorems about equivariant minimal surfaces in arbitrary non-positively curved symmetric spaces extending classical results on minimal surfaces in Euclidean space. First, we show that a complete equivariant branched immersed minimal surface in a nonpositively curved symmetric space of finite total curvature must be of finite Morse index. It is a generalization of the theorem by Fischer-Colbrie, Gulliver-Lawson, and Nayatani for complete minimal surfaces in Euclidean space.

Secondly, we show that a complete equivariant minimal surface in a nonpositively curved symmetric space is of finite total curvature if and only if it arises from a wild harmonic bundle over a compact Riemann surface with finite punctures. Moreover, we deduce the Jorge-Meeks type formula of the total curvature and show it is an integer multiple of  $2\pi/N$  for  $N$  only depending on the symmetric space. It is a generalization of the theorem by Chern-Osserman for complete minimal surfaces in Euclidean  $n$ -space.

This is joint work with Takuro Mochizuki (RIMS).

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**Speaker:** Song Dai (Tianjin University)

**Title:** Existence of harmonic metrics on nilpotent Higgs bundles over noncompact Riemann surfaces

**Abstract:** In this talk, we will first introduce the notions of Higgs bundles and harmonic metrics. Then we will survey some known results on the existence of harmonic metrics over noncompact Riemann surfaces. Our new result is that given a generically regular nilpotent harmonic bundle, there exists a (unique) maximal harmonic metric on the corresponding graded Higgs bundle. We will sketch the proof and show some applications.

This is a joint work with Qiongling Li.

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