Maths 260 Lecture 29

- **Topic for today:** Periodic orbits
- **Reading for this lecture:** None
- **Suggested exercises:** None
- **Reading for next lecture:** BDH Section 3.6
- **Today's handouts:** None

Example 1:

Sketch the phase portrait for the system

$$\frac{dx}{dt} = -y$$
$$\frac{dy}{dt} = 1 - 0.9y - x^2 - xy$$

Step 1: find equilibria:

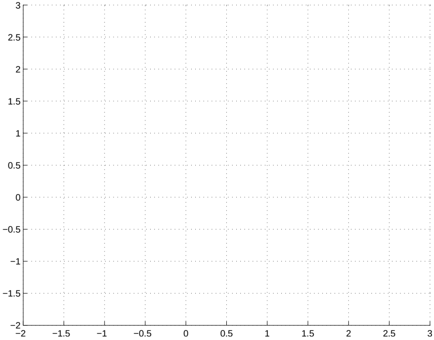
Jacobian:

Behaviour of solutions near (1,0)

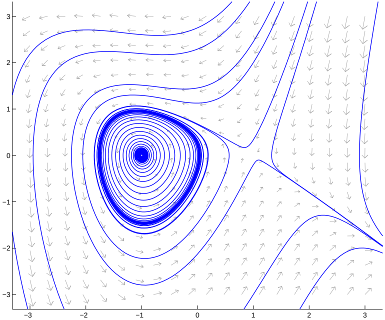
Behaviour of solutions near (-1,0)

Step 2: Find nullclines

$$\dot{x} = 0 \Rightarrow y = 0$$
$$\dot{y} = 0 \Rightarrow 1 - 0.9y - x^2 - xy = 0$$

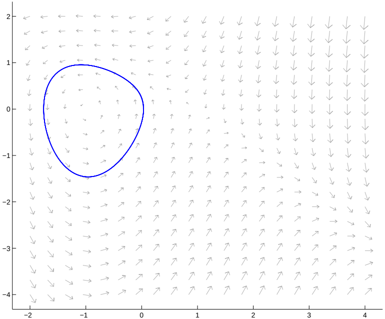


Phase portrait from pplane:



Note how close together solution curves get in a band around the equilibrium at $(-1, 0)$.

Careful use of pplane gives the following solution curve:



We see there is a closed solution curve passing close to the origin. This curve corresponds to a periodic solution of the system, i.e. a solution for which each dependent variable is a periodic function of time.

Example 2

Use pplane to investigate the qualitative changes in the behaviour of solutions to the system

$$\frac{dx}{dt} = -y$$
$$\frac{dy}{dt} = \lambda - 0.9y - x^2 - xy$$

that occur as λ is varied in the interval $[-3, 3]$.

Some advanced features of pplane are useful for investigating this system. In particular, pplane can be used to do the following.

- Plot nullclines. Select the Nullclines option in the lower right of the Setup window.
- Find equilibria and determine their type. Select the option Find an equilibrium from the Solutions menu on the Display window, move the cursor to a place in the display window near where you expect the equilibrium to be and click.
- Plot solutions for t increasing only. In the Display window, pull down the Options menu, pick Solution direction and then select Forward.
- Find a periodic solution. Select Find a nearly closed orbit and forward from the Solutions menu, move the cursor to a place in the display window near where you expect the periodic orbit to be, and click.

Important ideas from today:

- Autonomous systems with two or more dependent variables can have periodic solutions, where each of the dependent variables is a periodic function of the independent variable.
- A periodic solution lies on a closed curve in the phase plane.
- Nonlinear systems can exhibit many interesting bifurcations and, if there are three or more dependent variables, can exhibit a type of complicated behaviour called chaos. The study of bifurcations in nonlinear systems is called Dynamical Systems and is studied more in the courses Maths 363 and Maths 761.