DEPARTMENT OF MATHEMATICS

MATHS 260 **Tutorial 11**

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1. (a) Find the general solution of the following equations. Express your answers in terms of real-valued functions.

   i. $\dfrac{d^2y}{dt^2} + 2\dfrac{dy}{dt} - 2y = 0.$

   ii. $\dfrac{d^2y}{dt^2} + 4\dfrac{dy}{dt} + 3y = 0.$

   iii. $\dfrac{d^3y}{dt^3} + 4\dfrac{dy}{dt} = 0.$

   (b) For equation (i), describe the long term behaviour of the solution that satisfies each of the following initial conditions:

   i. $y(0) = 1, y'(0) = 6$

   ii. $y(0) = 1, y'(0) = -4$

   (c) Check your answers to (b) using *pplane*. To do this you must first convert the second order equation (i) to a system of first order equations.

2. A mass is sitting on a table, attached to a spring which is attached to a wall. The differential equation used to model the position of the mass can be written as

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = 0. \qquad (1)$$

   (a) Write the differential equation as a system of two first order differential equations.

   (b) Use `pplane` to investigate the behaviour of the solution to the system for the choices of m, b and k given below. Specifically, for each choice of the constants:

   - Determine the type of the equilibrium at the origin.
   - Look at the graphs of y and y' (use the 'Graph' menu in the display window, choose 'both' then click on the solution curve for which you want the y and y' graphs).
   - Hence describe the behaviour of the mass predicted by model (1).

   i. $b = 0, m = 10, k = 8.1$

   ii. $b = 4, m = 10, k = 8.1$

   iii. $b = 30, m = 10, k = 8.1$

   iv. $b = 18, m = 10, k = 8.1$

3. **Challenge question:**

   Determine the long term behaviour of the solution to the IVP

$$\frac{d^2y}{dt^2} + 0.2\frac{dy}{dt} - y + y^3 = 0, \quad y(0) = 0, \frac{dy}{dt}(0) = -1.$$