DEPARTMENT OF MATHEMATICS

MATHS 260 **Tutorial 4**

The aim of this tutorial is to get a better understanding of phase lines and their uses.

1. Consider the differential equation

$$\frac{dy}{dt} = y(a + y^2),$$

where a is a constant.

   (a) Use *dfield* to show the slope field for the case $a = -3$. Use the slope field to draw the phase line (by hand).

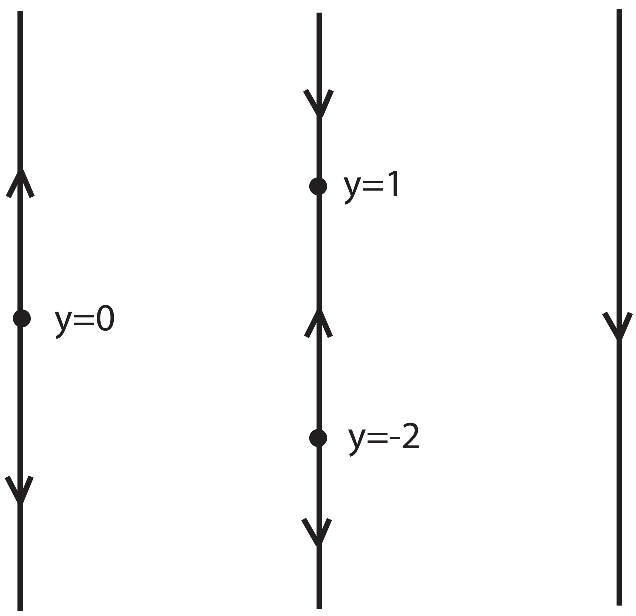
   (b) Now repeat (a) for the case $a = 3$.

2. For each of the following phase lines:

   (a) describe the long term behaviour of solutions;

   (b) write down a differential equation that would have the phase line shown.

   (c) check your answer to (b) using Matlab.



3. Consider the one-parameter family of differential equations

$$\frac{dy}{dt} = y(k - y),$$

where k is the parameter.

   (a) Draw the phase line (without using Matlab) for the cases $k = -1$, $k = 0$ and $k = 1$.

   (b) Sketch the bifurcation diagram.

(c) Use *dfield* to check your answers to parts (a) and (b).

(d) Write down what it means when we say two phase lines are "qualitatively the same".

(e) For what values of a are the phase lines for this DE qualitatively the same? At what value of a does the phase line undergo a qualitative change?

4. **Challenge question**
   Below is the bifurcation diagram for a first order differential equation.

   (a) Write down a first order differential equation which would have this bifurcation diagram.

   (b) Find bifurcation values of k, i.e., values of k where a change in the qualitative behaviour of solutions occurs.

   (c) Use the bifurcation diagram to predict the long term behaviour of the solution if:

      i. $k = -2, \quad y(0) = 0$
      ii. $k = 2, \quad y(0) = 1$
      iii. $k = 5, \quad y(0) = 3$