DEPARTMENT OF MATHEMATICS

MATHS 260 **Tutorial 6**

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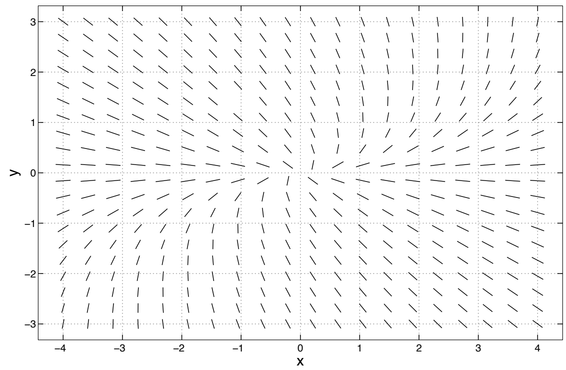
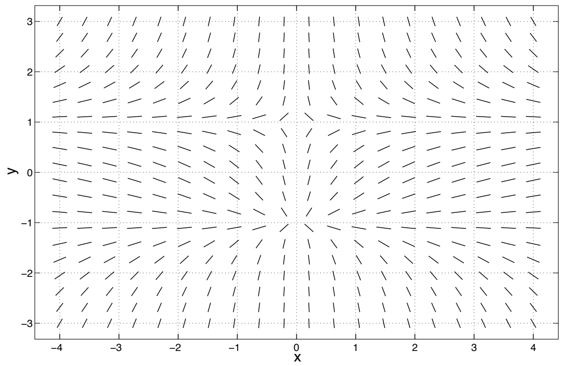
1. Without using Matlab, match the direction fields shown below with one of the following systems of DEs. Give reasons for your answers.

   Hint: first find where $dx/dt = 0$ and $dy/dt = 0$ on the pictures and use this to find the equilibrium solutions for each system. Compare your results with the direction fields shown.





   (a) $\dfrac{dx}{dt} = x, \quad \dfrac{dy}{dt} = y^2 + 1$

   (b) $\dfrac{dx}{dt} = x, \quad \dfrac{dy}{dt} = x + y$

   (c) $\dfrac{dx}{dt} = x - y, \quad \dfrac{dy}{dt} = 2y$

   (d) $\dfrac{dx}{dt} = -x, \quad \dfrac{dy}{dt} = y^2 - 1$

2. (a) Without using Matlab, find the equilibrium solutions of the system of equations

$$\frac{dx}{dt} = x + y,$$
$$\frac{dy}{dt} = x - x^3 - y.$$

   (b) Use *pplane* to draw the direction field and to find the equilibrium solutions. This is done under the `Solutions` menu - choose `Find an equilibrium point` then click the cursor near a possible equilibrium point. Do these equilibrium points agree with your answers found in (a)?

3. Consider the following system of equations

$$\frac{dx}{dt} = x - 2y,$$
$$\frac{dy}{dt} = -2y.$$

   (a) Use *pplane* to plot the direction field and some solutions to this system of equations.

   (b) Use the direction field to locate any straightline solutions to the system. For any straightline solutions that you find, try to guess a simple formula for the solution based on what you see in the phase portrait.

(c) Rewrite the system above in vector form, i.e., find matrix **A** such that

$$\frac{d\mathbf{Y}}{dt} = \mathbf{A}\mathbf{Y},$$

where

$$\mathbf{Y} = \begin{pmatrix} x \\ y \end{pmatrix}.$$

(d) Find the eigenvalues and eigenvectors of **A** and hence write down formulae for the straightline solutions of the system of equations. Compare your answers with those you got in (b) above. Explain any differences you see.

(e) Use your answer to (d) to write down the general solution to the system of equations. Hence find the solution that satisfies the initial condition

$$\mathbf{Y}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

(f) Use *analyzer* to plot your solution from (e) (i.e., the functions $x(t)$ and $y(t)$ that you find). Then use *pplane* to plot the functions $x(t)$, $y(t)$ for the same solution. This is done under the `Graph` menu - choose `Both` - then click the cursor on the solution you wish to plot. Compare the plots from *analyzer* and *pplane* and comment on the similarities and differences you see.

4. **Challenge question:**

(a) Using any methods you can, locate the straightline solutions of the system of equations

$$\begin{aligned} \frac{dx}{dt} &= -x - y \\ \frac{dy}{dt} &= -y. \end{aligned}$$

(b) Can you write the general solution to the DE as a linear combination of straightline solutions? What goes wrong?

(c) Use *pplane* to draw a phase portrait for this system, including any straightline solutions you found in (a). Can you see any clues in the phase portrait which might explain the difficulties in finding the solution to the IVP in (b)?