DEPARTMENT OF MATHEMATICS

MATHS 260 **Tutorial 9**

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1. Consider the linear system
$$\frac{dY}{dt} = AY,$$
where A is as given below. For each choice of A:

   - Sketch the phase portrait.
   - Use `pplane` to draw the phase portrait and compare with your sketch.
   - Describe the long term behaviour of the solutions.
   - Write down the general solution of the system in terms of real-valued functions.

   (a) $A = \begin{pmatrix} -2 & -1 \\ 1 & -4 \end{pmatrix}$. Eigenvalues are -3, -3.
   Eigenvector is $(1, 1)^T$.

   (b) $A = \begin{pmatrix} 1 & 2 \\ -2 & -4 \end{pmatrix}$. Eigenvalues are 0 and -3.
   Eigenvectors are $(2, -1)^T$ and $(-1, 2)^T$ respectively.

2. Consider the following nonlinear system of equations:

$$\frac{dx}{dt} = x - y$$
$$\frac{dy}{dt} = x(y^2 - 1)$$

   Work out your answers by hand for parts (a) and (b).

   (a) Find all equilibria and determine their types.

   (b) For each equilibria, sketch a phase portrait showing the behaviour of solutions in the associated linearised system.

   (c) Check your answers using `pplane`.

[Final question over page]

3. **Challenge question:**

Use `pplane` to investigate the following nonlinear system as the parameter α changes.

$$\frac{dx}{dt} = \alpha x + y + \sin(x)$$
$$\frac{dy}{dt} = x - y$$

(a) For what range of α is there more than one equilibrium?
Hint: The behaviour of the system is more interesting for $\alpha < 0$ than $\alpha > 0$! Try starting with the range $-4 < \alpha < 1$.

(b) Once you have seen how the system behaves, you may find it useful to consider what happens to $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 0$ as α is varied.
Can you describe what is happening?