Department of Mathematics

MATHS 260 Differential Equations

Mid-Semester Test

Wednesday, 20 September, 2006

Instructions

- This test contains **FIVE** questions. Attempt **ALL** questions.

- The total is **50 marks**.
- Show **ALL** your working.

- You have **60 minutes** to do the test.

1. (10 marks)

   (a) (5 marks) Find the solution to the initial value problem

   $$\frac{dy}{dt} = 3t - \frac{y}{t}, \quad y(1) = 2.$$

   (b) (5 marks) Find the general solution to the following differential equation

   $$\frac{dy}{dt} = e^{2t}\sqrt{y}.$$

2. (10 marks)
   Consider the following initial value problem

   $$\frac{dy}{dt} = y + 2t, \ y(0) = 1.$$

   (a) (3 marks) Does a unique solution of the IVP exist? Give reasons for your answer.

   (b) (7 marks) Use Improved Euler with stepsize $h = 0.5$ to find an approximation to the solution at $t = 1$.

3. (8 marks)
   A Runge-Kutta method was used with the stepsizes $h = 0.05$ and $h = 0.025$ to approximate the solution of a differential equation. When comparing with the exact solution, the following results were found:

h	Approximation	Error
0.05	7.389044767	1.1332×10^{-5}
0.025	7.389055360	7.3830×10^{-7}

   (a) (5 marks) What do you think the order of the method is? Give reasons for your answer.

   (b) (3 marks) If the Runge-Kutta method was used with stepsize $h = 0.0125$, estimate the error in the approximation. Give reasons for your answer.

4. (10 marks)
   Consider following differential equation

$$\frac{dy}{dt} = y(y^2 - \mu + 1).$$

   (a) (6 marks) Find all equilibrium solutions and determine their types (e.g., source, node).

   (b) (4 marks) Draw the bifurcation diagram. Identify any values of μ for which a bifurcation exists.

5. (12 marks)
   Consider the following system of differential equations:

$$\frac{dY}{dt} = \begin{pmatrix} -3 & 1 \\ 0 & -2 \end{pmatrix} Y$$

   where $Y = (x, y)^T$.

   (a) (4 marks) Find a solution to this system of equations. Your answer should contain two arbitrary constants.

   (b) (3 marks) Find the solution that passes through $(x, y) = (0, 2)$ at $t = 0$. Express your solution in the form $(x(t), y(t))$.

   (c) (5 marks) Sketch the phase portrait showing:

   - all equilibrium solutions;
   - all straight line solutions;
   - the solution you found in part (b) above;
   - three other solution curves.

   For each solution curve you draw, you should draw an arrow indicating the direction moved as time increases.