Maths 260 Overview

A differential equation expresses a relationship between a function and its derivatives (or some functions and their derivatives, in the case of a system of equations). For example, the DE

$$\frac{dx}{dt} = 3x$$

says the unknown function $x(t)$ is equal to one third of its derivative. Differential equations are useful for modelling many different physical phenomena, e.g., motion of bridges, growth of populations, chemical reactions.

The main aim of this course is to learn ways to determine the properties of the functions that satisfy a given DE, i.e., properties of solutions to a DE. Information we seek about solutions may be:

- qualitative information about solutions;
- an explicit formula for a solution;
- a numerical approximation to an explicit formula.

Before seeking a solution or information about a solution, we would like to be sure that a solution to the DE exists. The Existence and Uniqueness Theorem gives conditions under which solutions to a DE (or system of DEs) exist. These conditions turn out to be fairly general — solutions exist for a large class of DEs.

Finding Explicit Formulae for Solutions

Although the Existence and Uniqueness Theorem guarantees that solutions exist for many DEs, it is not usually possible to write down a formula for a solution to a DE. Special cases in which it is sometimes possible to find formulae for solutions include:

- separable or linear DEs with one dependent variable;
- linear, constant coefficient, autonomous systems of first order DEs;
- linear, constant coefficient, higher order DEs, either homogeneous or nonhomogeneous.

Qualitative Information

A major technique for getting qualitative information about solutions to autonomous systems is to sketch the phase line or phase plane. This can involve:

- finding equilibrium solutions;
- classifying equilibrium solutions using linearisation or some other method;
- using the slope field and/or nullclines.

In DEs that depend on a parameter, the qualitative behaviour of solutions may change as the parameter varies. A bifurcation diagram can be useful for summarising the qualitative changes in the behaviour of solutions that occur as a parameter is varied.

Numerical Methods

Numerical approximations to explicit solutions at particular times can be found using a numerical integration method. We looked briefly at fixed stepsize methods: Euler's method, Improved Euler's method, and 4th order Runge-Kutta method. Numerical methods can also be used to draw approximate phase portraits. We used the software package Matlab for this purpose.

A thorough investigation of the behaviour of solutions to a differential equation will usually involve using more than one type of method (e.g., both qualitative and numerical methods).

Where to from here?

If you have enjoyed Maths 260 and want to do more courses in Applied Maths, consider:

- Maths 162: Computation and Modelling
- Maths 270: Numerical Computation
  Prerequisite: Maths 108/150 and Maths 162 (or equivalent). This is a required course for the major in Applied Maths.
- Maths 253: Advancing Mathematics 3
  Prerequisite: Maths 250 or A pass in Maths 208. This is a prerequisite for all Stage 3 Applied Maths papers.

- Maths 361: Partial Differential Equations
  Prerequisites: Maths 260 and Maths 253. This is a required course for the major in Applied Maths.
- Maths 340: Real and Complex Calculus
  Prerequisite: Maths 253. This is a required course for the major in Applied Maths.
- Maths 362: Methods in Applied Maths
  Prerequisite: Maths 260 and Maths 253. Recommended prior pass in Maths 361 and Maths 340.
- Maths 363: Advanced Modelling and Computation
  Prerequisite: Maths 260 and Maths 270. A major in Applied Maths requires either Maths 362 or Maths 363.

For help in selecting courses, see the Mathematics Department Undergraduate Handbook or speak to your lecturers.

Preparing for the Final Exam

To do well on the exam, you need to:

- understand the lecture material. Make sure you have a complete set of lecture notes and handouts. The textbook has good explanations of most topics and should be read in conjunction with your lecture notes.
- practise doing examples. Good sources of examples are old exams and tests, assignment and tutorial questions, and exercises in the text book.

Relevant material in the textbook is contained in: Chapter 1, sections 1.1-1.9, Chapter 2, sections 2.1-2.4, Chapter 3, sections 3.1-3.8, Chapter 4, sections 4.1-4.4, Chapter 5, sections 5.1-5.2, Chapter 7, sections 7.1-7.3.

Exam technique

- Bring your calculator.
- The exam is two hours long.
- Total marks $=100$.
- 100 marks in 120 minutes means you should allow one minute per mark.

  For example, for a 25 mark question, spend 25 minutes on the question then MOVE ON.

  There should be a few minutes left at the end to go back to any unfinished questions.
- It is usually easier to get the first half of the marks for a question than the last half of the marks, so try all questions.
- Get a good night's sleep before the exam.

Getting help before the exam

- Written answers to old exams and tutorials will not be distributed. Written answers to assignments are on the website.
- You can get some help from your lecturers before the exam. Office hours before the exam are:
  - Vivien: Friday Oct 30th, 10-12 and Monday Nov 2nd, 12-2.
  - Amanda: Thursday Oct 29th, 10-12 and Monday Nov 2nd, 10-12.
  - Claire: to be announced (see the website)

  If these times are not convenient, email one of your lecturers to make an appointment.