Department of Mathematics

MATHS 260 Differential Equations
Mid-semester Test, Wednesday, May 7th, 2008

This test contains **FIVE** questions. Attempt **ALL** questions. Show **ALL** your working. You have 50 minutes to do the test. Total marks = 45.

1. (8 marks) Consider the differential equation

$$\frac{dy}{dt} = 3t^2y^2.$$

   (a) Show that

$$y_1(t) = -\frac{1}{t^3}$$

   is a solution to the differential equation.

   (b) Find a one-parameter family of solutions to the differential equation.

   (c) Find a solution to the differential equation that also satisfies the initial condition $y(0) = 1$.

2. (16 marks) Consider the one-parameter family of differential equations

$$\frac{dy}{dt} = \mu + 2y + y^2.$$

   (a) For the case $\mu = 0$, find all equilibrium solutions and determine their type (e.g., sink, source). Sketch the phase line.

   (b) Repeat (a) for the case $\mu = 1$.

   (c) Repeat (a) for the case $\mu = 2$.

   (d) Now let μ vary. Locate the equilibrium solutions and determine their type. Sketch the bifurcation diagram. Be sure to label the main features of the bifurcation diagram.

3. (8 marks) Consider the system of equations

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} a & 1 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix},$$

   where a is a constant.

   (a) Find all values of a for which the equilibrium at the origin is a saddle.

   (b) For the case $a = 1$, find the general solution to the system of equations and sketch the phase portrait.

4. (7 marks) Consider the following initial value problem:

$$\frac{dy}{dt} = ty + t + y, \quad y(1) = 0.5.$$

(a) Use two steps of Euler's method to compute an approximation to the solution of the initial value problem at final time $t = 3$. Show all your working.

(b) State two changes you could make to the procedure you used in (a) that might give you a better approximation to the solution to the initial value problem.

(c) If you were using a numerical method on a computer to calculate an approximate solution, how could you check whether the method you were using was accurate enough?

5. (6 marks) The following equation is proposed as a model of the population of possums living in Albert Park:

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{50}\right)\left(\frac{P}{20} - 1\right) - E,$$

where P measures the possum population ($P = 1$ means that there are 100 possums in the Park), and E is a parameter measuring the rate of emigration of the possums out of the Park.

Briefly describe (i.e., in two or three sentences) the methods you could use to get information about solutions to this model. You do not need to do any calculations to answer this question.