DEPARTMENT OF MATHEMATICS

MATHS 260 **Tutorial 2**

The aim of this tutorial is to learn to use the functions *dfield* and *numerical* from Matlab, and to use these functions to find out about solutions to some differential equations.

1. Learn to use the Matlab function *dfield* by working through the procedure given on the handout "An introduction to software used in Maths 260".

2. Learn to use the Matlab function *numerical* by working through the procedure given on the handout "An introduction to software used in Maths 260".

3. Consider the differential equation

$$\frac{dy}{dt} = 2y + t, \quad y(-2) = 1.$$

   (a) Use *dfield* to plot a slope field for the differential equation, for $t \in [-2, 10]$.

   (b) Choose `Options/Solver/Euler`. When a window pops up, change the stepsize to 2. Don't forget to select the `Change settings` button.

   (c) Choose `Options/Keyboard input`. Put the initial condition in the pop up window and select `Compute`.

   - Notice how the slope of the numerical solution matches the slope of the slope field at the beginning of the step.
   - Use the slope field to estimate the value of $y(0)$ that you would get if you used Euler's method with stepsize $h = 2$ to approximate the solution of the initial value problem at the final $t = 0$.

   (d) Do a pencil and paper calculation with Euler's method with $h = 1.0$ to work out the approximate value of the solution to the initial value problem at final $t = 0$. Check your answer by changing the stepsize in *dfield* to 1.

   (e) The solution to the initial value problem is

$$y(t) = -\frac{t}{2} - \frac{1}{4} + \frac{e^{2t+4}}{4}.$$

   i. Check this is true by substitution.

   ii. Use the expression for $y(t)$ to calculate the value of the solution to the IVP at $t = 0$.

   iii. Hence show that the error in the approximation you obtained in (d) is greater than 11(this is a huge error!)

   iv. How could you adapt your method in (d) to obtain a more accurate approximation to the initial value problem?

(f) Use *numerical* to estimate $y(0)$ using Euler's method. For the "solution at final t", use `(exp(4)-1)/4`.

   i. What do you notice about the errors shown?

   ii. What do you notice about the effective order shown?

(g) Repeat (f) for Improved Euler and 4th order Runge-Kutta.

4. **Challenge question**: Use *dfield* to investigate the behaviour of solutions to the differential equation

$$\frac{dx}{dt} = x(1-x) + 0.1 \sin t.$$

Look for initial conditions that lead to qualitatively different types of long term behaviour for solutions. You may need to change the scaling of the t-axis to work out what is happening for large t.

Experiment with the size of the coefficient of the $\sin t$ term to see if this changes the qualitative behaviour of solutions. What is going on?