DEPARTMENT OF MATHEMATICS

MATHS 260 Tutorial 5

The aim of this tutorial is to practice solving linear equations and to start thinking about systems of equations.

1. Consider the differential equation

$$\frac{dy}{dt} = -2y + t.$$

   (a) Use one of the methods discussed in Lectures 12 and 13 to find the general solution.

   (b) Use *analyzer* to plot some of the solutions found in (a).

   (c) Use *dfield* to plot the slope field and some approximate solutions to the DE. Try to plot the same solutions as you plotted in (b).

   (d) Compare the pictures you plotted in (b) and (c). Are they exactly the same as each other? If so, say why this is so. If not, explain why not.

   (e) Explain your answer to (d) to your tutor.

2. (a) Solve the initial value problem

$$\frac{dy}{dt} + y = \cos 3t, \quad y(0) = 0.$$

   (b) Check your answer to (a) but substituting into the differential equation.

   (c) Describe the long term behaviour of the solution you found in (a).

   (d) Use *dfield* to check your answer to (c).

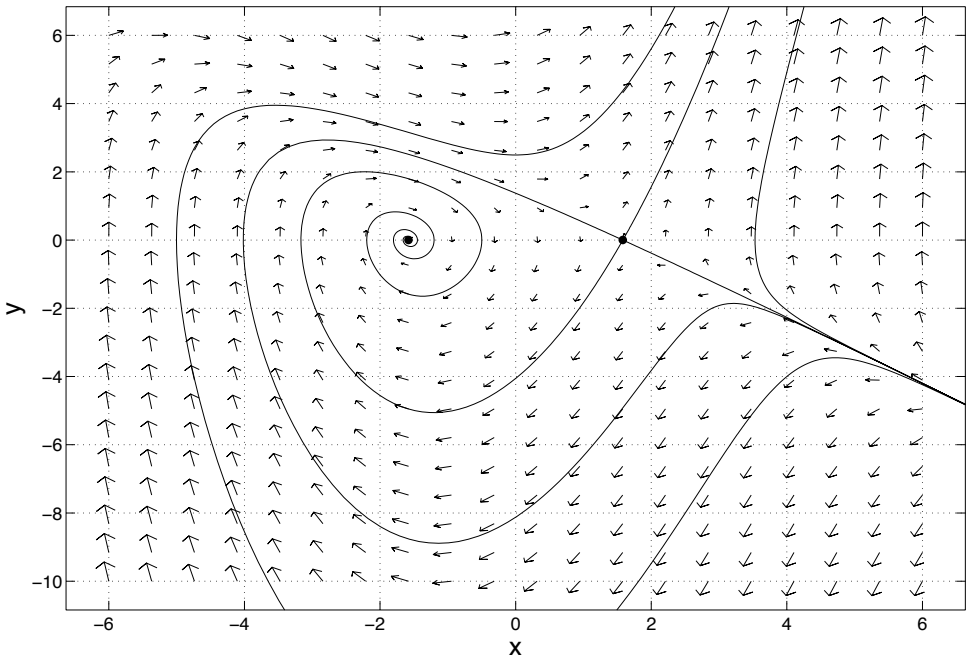
3. Learn to use the Matlab function *pplane* by working through the example given on the handout "An introduction to software used in Maths 260".

(More questions over page)

4. Based on what you see in the following phase portrait, describe the long term behaviour of solutions through the points:

   (i) $(x(0), y(0)) = (0, 0)$ (ii) $(x(0), y(0)) = (4, 4)$

   (iii) $(x(0), y(0)) = (4, -4)$ (iv) $(x(0), y(0)) = (0, -6)$.



5. **Challenge question:** Find some bifurcation values of k for the system of equations

$$\begin{aligned} \frac{dx}{dt} &= -y, \\ \frac{dy}{dt} &= k - x^2 - 0.9y - xy. \end{aligned}$$

   Hint: there are at least three bifurcation values for $k \in [-3, 3]$.