Maths 260 Lecture 21

- **Topics for today:**
  Complex eigenvalues and eigenvectors

- **Reading for this lecture:**
  Some notes on complex numbers
  BDH Appendix C

- **Suggested exercises:**
  Problems at the back of "Some notes on complex numbers"
- **Today's handouts**: Tutorial 8

Complex eigenvalues and eigenvectors

The procedure for finding complex eigenvalues and eigenvectors for a matrix is the same as for real eigenvalues and eigenvectors but the calculations can seem trickier because of the complex algebra.

Example 1:

Find the determinant of the matrix $\begin{pmatrix} 4-\lambda & -1 \\ 3 & 2-\lambda \end{pmatrix}$

Example 2

Find the determinant of the matrix $\begin{pmatrix} -1-\lambda & 0 & 0 \\ 2 & 1-\lambda & 0 \\ -2 & 3 & 7-\lambda \end{pmatrix}$

Example 3

Find the eigenvalues and eigenvectors of the matrix $\begin{pmatrix} 3 & 2 \\ -2 & 3 \end{pmatrix}$.

Example 4

Find the eigenvalues and eigenvectors of the matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & -3 \\ 1 & 3 & 2 \end{pmatrix}.$$

Examples 3 and 4 illustrate two important points:

- If a matrix has only real entries, then any complex eigenvalues come in complex conjugate pairs.
- The corresponding eigenvectors come in complex conjugate pairs too.

- It is not always obvious when a vector with complex entries is a constant multiple of another vector - but it is easy to check by multiplication.

Example 5: Show that $\begin{pmatrix} 2i \\ -1 \end{pmatrix}$ is an eigenvector of the matrix $\begin{pmatrix} 1 & 4 \\ -1 & 1 \end{pmatrix}$ corresponding to the eigenvalue $1 + 2i$.

Example 6

Show that $\begin{pmatrix} 3i \\ 1 \\ 0 \end{pmatrix}$ is an eigenvector of the matrix

$\begin{pmatrix} 1 & 9 & 1 \\ -1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ corresponding to the eigenvalue $1 - 3i$.

- We often want to compute the real and imaginary parts of complex-valued expressions.

Example 7:

Compute the real and imaginary parts of $e^{(2-i)t}\begin{pmatrix} 2+i \\ 3 \end{pmatrix}$

Example 8

Compute the real and imaginary parts of $e^{(2+3i)t}\begin{pmatrix} 1-3i \\ 4i \end{pmatrix}$

Example 9

Compute the real and imaginary parts of $e^{-3t}\begin{pmatrix} -i \\ 3 \\ 3i-4 \end{pmatrix}$

Important ideas from today's lecture:

- Computing complex eigenvalues and eigenvectors
  - If a matrix has only real entries, then any complex eigenvalues come in complex conjugate pairs
  - The corresponding eigenvectors come in complex conjugate pairs too
- Separating real and imaginary parts of complex-valued expressions