Maths 260 Lecture 21

Topics for today:

Complex eigenvalues and eigenvectors

 Reading for this lecture: Some notes on complex numbers BDH Appendix C

Suggested exercises:

Problems at the back of "Some notes on complex numbers"

Today's handouts: Tutorial 8

Complex eigenvalues and eigenvectors

The procedure for finding complex eigenvalues and eigenvectors for a matrix is the same as for real eigenvalues and eigenvectors but the calculations can seem trickier because of the complex algebra.

Example 1:

Find the determinant of the matrix

$$\left(egin{array}{cc} 4-\lambda & -1 \ 3 & 2-\lambda \end{array}
ight)$$

Find the determinant of the matrix

$$\left(egin{array}{ccc} -1-\lambda & 0 & 0 \ 2 & 1-\lambda & 0 \ -2 & 3 & 7-\lambda \end{array}
ight)$$

Find the eigenvalues and eigenvectors of the matrix $\begin{pmatrix} 3 & 2 \\ -2 & 3 \end{pmatrix}$.

Find the eigenvalues and eigenvectors of the matrix

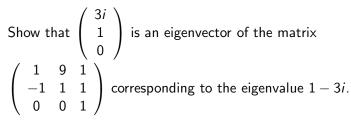
$$\left(\begin{array}{rrrr} 1 & 0 & 0 \\ 0 & 2 & -3 \\ 1 & 3 & 2 \end{array}\right).$$

Examples 3 and 4 illustrate two important points:

- If a matrix has only real entries, then any complex eigenvalues come in complex conjugate pairs.
- The corresponding eigenvectors come in complex conjugate pairs too.

It is not always obvious when a vector with complex entries is a constant multiple of another vector - but it is easy to check by multiplication.

Example 5: Show that $\begin{pmatrix} 2i \\ -1 \end{pmatrix}$ is an eigenvector of the matrix $\begin{pmatrix} 1 & 4 \\ -1 & 1 \end{pmatrix}$ corresponding to the eigenvalue 1 + 2i.



 We often want to compute the real and imaginary parts of complex-valued expressions.

Example 7:

Compute the real and imaginary parts of $e^{(2-i)t} \begin{pmatrix} 2+i\\ 3 \end{pmatrix}$

Compute the real and imaginary parts of $e^{(2+3i)t} \begin{pmatrix} 1-3i\\4i \end{pmatrix}$

Compute the real and imaginary parts of $e^{-3t} \begin{pmatrix} -i \\ 3 \\ 3i - 4 \end{pmatrix}$

Important ideas from today's lecture:

- Computing complex eigenvalues and eigenvectors
 - If a matrix has only real entries, then any complex eigenvalues come in complex conjugate pairs
 - The corresponding eigenvectors come in complex conjugate pairs too

 Separating real and imaginary parts of complex-valued expressions