Maths 260 Assignment 4

September 30, 2009 Due: 4pm, Thursday 15 October 2009

- Students should hand their assignments in to the correct box in the basement of the Mathematics/Physics building **before** 4pm on the date due.
- Late assignments or assignments placed in the wrong box will not be accepted.
- Your assignment **must** be accompanied by a blue Mathematics Department coversheet. Copies of the coversheet are available in the basement.

1. Find the general solution to the linear equation

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix} \mathbf{Y}$$

Hence find the solution which satisfies $\mathbf{Y} = (1, 1)$ at $t = 0$.

2. Consider the one-parameter family of linear systems

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} a & -2 \\ 1 & 0 \end{pmatrix} \mathbf{Y}$$

where a is a parameter. Determine the type of equilibrium at the origin for all values of a. Sketch the phase portraits for representative values of a, including those values of a at the boundaries between regions.

Hint: you should find 4 different regions. Show all your working.

3. Consider the following system of equations:

$$\frac{dx}{dt} = x(2x - y)$$
$$\frac{dy}{dt} = y(x^2 - 4)$$

(a) Find all equilibrium solutions and determine their type. For each equilibrium solution, sketch a phase portrait showing the behaviour of solutions very near to that equilibrium.

(b) Find the nullclines and determine the direction in which solutions move on and between all nullclines.

(c) Using your answers to the above, carefully draw the phase portrait for this system. You may use Matlab to check your answers, but you must show your working for any results that you wish to be marked.

4. The outbreak of a disease such as influenza within a population can be modeled with the system of equations:

$$\begin{aligned}\frac{dS}{dt} &= -aSI,\\ \frac{dI}{dt} &= aSI - bI,\\ \frac{dR}{dt} &= bI,\end{aligned}$$

where a and b are positive constants. In this model $S(t)$ is the number of susceptible people in the population (these are people who have not had influenza and have no immunity to it). $I(t)$ is the number people in the population who are infected with influenza. $R(t)$ is the number of recovered people in the population (these are people who no longer have influenza and are now immune to it). In this model, nobody dies from the disease. The variable t is time.

(a) Briefly describe the physical significance of each term in the model.

(b) One way used to prevent the spread of influenza is by vaccinating susceptible people in the population. This works because people who get vaccinated become immune to the disease. Suggest how you could modify the model to include this effect.

(c) Briefly describe (i.e., in one or two paragraphs) some methods you could use to analyse these equations to get information about solutions to the model.

5. Each of the pictures below shows the direction field and some solutions for a system of nonlinear equations. Equilibrium solutions are marked with dots. On a copy of each picture, mark the approximate positions of all x-nullclines and all y-nullclines. Give reasons for your answers. Hand in this sheet with your answers.