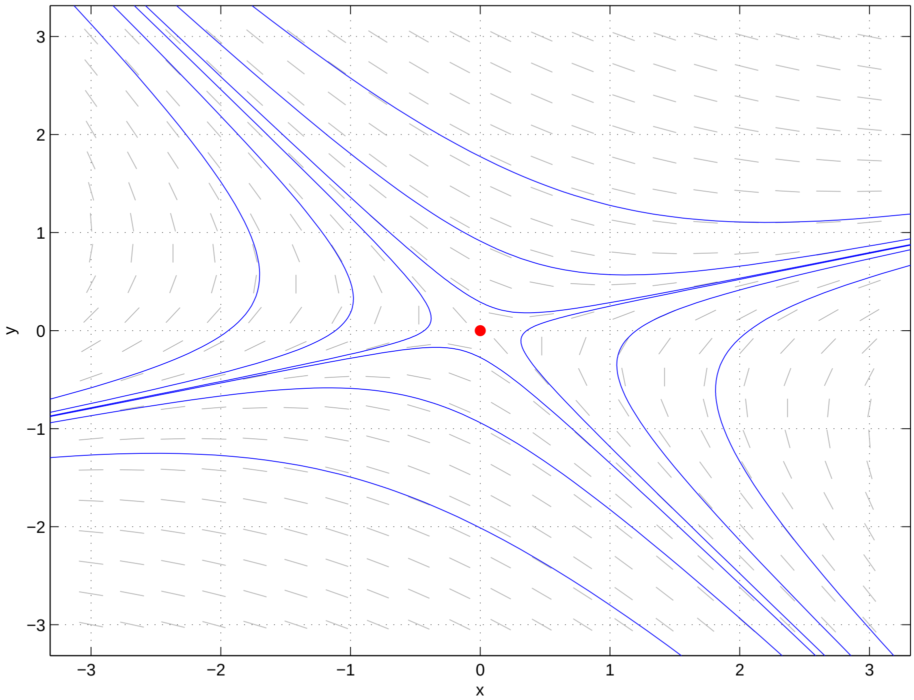
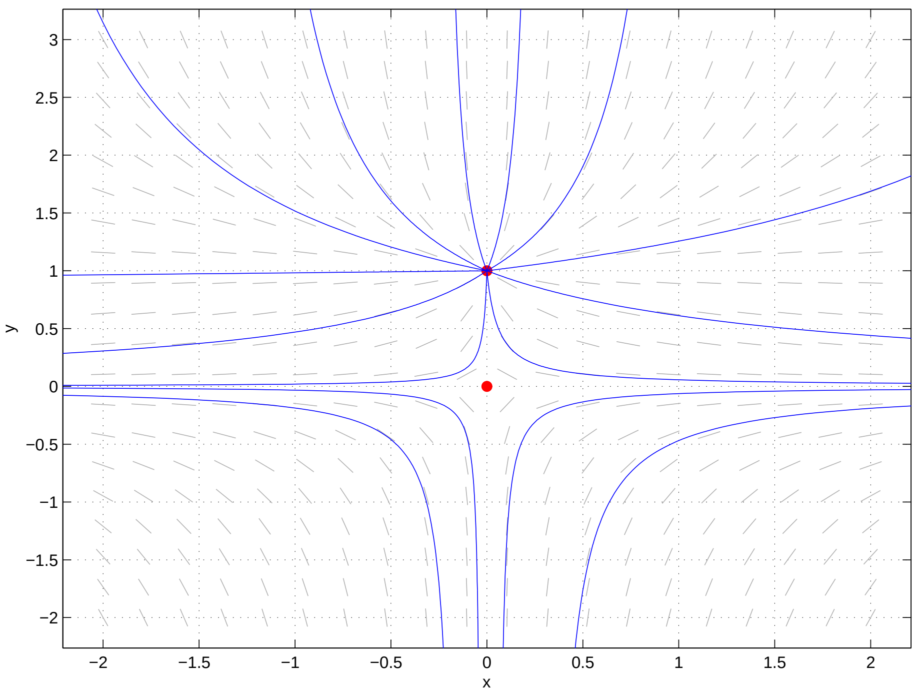
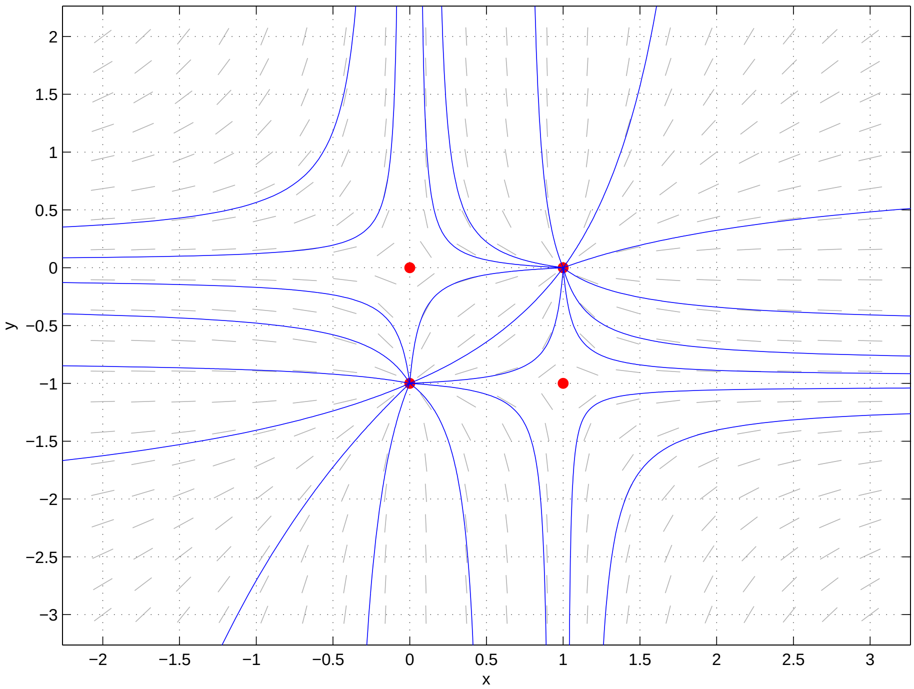
DEPARTMENT OF MATHEMATICS

MATHS 260 **Tutorial 10**

1. The pictures below show the direction field and some solutions for some systems of equations. Equilibrium solutions are marked with dots. On the pictures, mark the approximate positions of all x-nullclines and all y-nullclines. Give reasons for your answers.







2. Consider the nonlinear system of equations

$$\frac{dx}{dt} = 2y - y^2 - x$$
$$\frac{dy}{dt} = x.$$

(a) **Without using Matlab**, find all equilibrium solutions and determine their type. For each equilibrium solution, sketch a phase portrait showing the behaviour of solutions very near to that equilibrium.

(b) Find the nullclines, and determine the direction in which solutions move on and between all nullclines.

(c) Carefully draw the phase portrait for this system.

(d) Use `pplane` to check your answers. In particular:

i. Use `Solutions/Find an equilibrium point` to show the equilibria. Observe the behaviour near these points. Does it fit with your analysis of the Jacobian there? Do the eigenvalues computed by `pplane` match the values you calculated?

ii. Use `Solutions/Show nullclines` in `pplane` to add the nullclines to the phase portrait. Observe the direction arrows on the nullcllines. Observe the direction arrows in the regions between the nullclines. Does this match the directions that you calculated?

iii. Choose an initial point in each region and click to show the solution through that point. Note the slope of solutions as they cross the nullclines. Does the phase portrait match the one you sketched?

3. Two Martian species, the Zarks and the Yoiks, live in close proximity on Mars. They share the same basic requirements so they are in constant competition for resources. Such a situation can be modelled with the following system of equations:

$$\frac{dZ}{dt} = rZ(1 - aZ - bY)$$
$$\frac{dY}{dt} = sY(1 - cY - dZ)$$

where $r, s, a, b, c, d > 0$. Here, $Z(t)$ is the population of Zarks at time t and $Y(t)$ is the population of Yoiks at time t. For the model to be realistic, we must have $Z(t) \geq 0$ and $Y(t) \geq 0$ for all t.

(a) Briefly describe the physical significance of each term in the model.

(b) Briefly describe (in one or two paragraphs) how you would try to analyse these equations to find out what kind of solutions the model supports. Think about the methods we have used in class to try and understand these kind of systems.

(c) When we finally have a spaceship that will take us to Mars, hopefully we will see exactly what the population levels of Zarks and Yoiks are. Meantime, mathematicians know that this type of model can support four steady states.

One obvious steady state is where neither population exists. How can you tell this from looking at the equations?

What do you think the other three steady states are?