

Maths 260: Data used in Lecture 5

The following tables are used in this lecture and are provided here for your convenience.

Euler's method with various step sizes was used to approximate the solution to the IVP

$$\frac{dy}{dt} = y, \quad y(0) = 1,$$

at time $t=1$, and the following results were obtained. The error given is the difference between the approximation and the true value of the solution, which is $y(1)=e$.

| No. of Steps | $y(1)$ | error |
|--------------|----------|---------|
| 1 | 2.000000 | 0.718 |
| 2 | 2.250000 | 0.468 |
| 4 | 2.441406 | 0.277 |
| 8 | 2.565784 | 0.152 |
| 16 | 2.637928 | 0.0804 |
| 32 | 2.676990 | 0.0413 |
| 64 | 2.687345 | 0.0209 |
| 128 | 2.707739 | 0.0105 |
| 256 | 2.712992 | 0.00529 |
| 512 | 2.715632 | 0.00265 |

Looking at the same IVP with Improved Euler yields:

| No. of Steps | $y(1)$ | error |
|--------------|----------|------------|
| 1 | 2.500000 | 0.218 |
| 2 | 2.640625 | 0.0777 |
| 4 | 2.694856 | 0.0234 |
| 8 | 2.711841 | 0.00644 |
| 16 | 2.716594 | 0.00169 |
| 32 | 2.717850 | 0.000432 |
| 64 | 2.718173 | 0.000109 |
| 128 | 2.718254 | 0.0000274 |
| 256 | 2.718275 | 0.00000689 |
| 512 | 2.718280 | 0.00000173 |

The same IVP with RK4 yields:

| No. of Steps | $y(1)$ | error |
|--------------|----------|--------------|
| 1 | 2.708333 | 0.00994 |
| 2 | 2.717346 | 0.000936 |
| 4 | 2.718210 | 0.0000719 |
| 8 | 2.718277 | 0.00000498 |
| 16 | 2.718282 | 0.000000328 |
| 32 | 2.718282 | 0.0000000215 |

These numerical results were all obtained using the function `numerical.m` within Matlab.