Maths 260 Assignment 1

July 27, 2009 Due: 4pm, Tuesday, August 11, 2009

- Put your completed assignment in the appropriate box in the basement of the Maths/Physics building **before** 4pm on the date due.
- Late assignments or assignments placed in the wrong box will not be marked.
- Your assignment must be accompanied by a blue Mathematics Department coversheet. Copies of the coversheet are available from a box in the basement.

1. This question is about the differential equation

$$\frac{dy}{dt} = \frac{te^{-2t}}{y}.$$

(a) Find a one-parameter family of solutions to the differential equation (i.e., a formula for solutions, with one arbitrary constant in the formula).

(b) Are there any solutions to the differential equation that are missing from the set of solutions you found in (a)? Explain your answer.

(c) Find a solution to the differential equation that satisfies the initial condition $y(0) = 1$.

(d) Find a solution to the differential equation that satisfies the initial condition $y(0) = -2$.

(e) Use the Matlab function *analyzer* to draw three different solutions to the differential equation, including the solutions you found in parts (c) and (d). Make sure your solutions are easily distinguishable in the analyzer plot. Draw all three graphs on the same picture, print your picture, and hand it in with your assignment.
Note: One or more of your solutions may not be defined for all t but *analyzer* may still plot something for the values of t for which the solution is not defined. Make sure you indicate on your Matlab plots which parts (if any) are Matlab plotting errors.

2. (a) Consider the differential equation

$$\frac{dx}{dt} = -x^2 + t.$$

i. Use *dfield* to plot and print two copies of the direction field for this differential equation. Use the ranges $t \in [-2, 4]$ and $x \in [-3, 3]$.

ii. Use one copy of the direction field to sketch at least four representative solutions to the differential equation, including the solution that satisfies the initial condition $x(1) = -1$.

iii. Use the other copy of the direction field to show what would be obtained if Euler's method with stepsize $h = 1$ was used to compute an approximation at final time $t = 4$ to the solution that satisfies the initial condition $x(1) = -1$. You do not need to do any calculations to do this part of the question; just use the information on the direction field.

(b) Use Improved Euler's method with stepsize $h = 1$ to compute an approximate value of the solution to the initial value problem

$$\frac{dx}{dt} = -x^2 + t, \; x(1) = -1$$

at final time $t = 4$. Show all your working.

(c) A numerical method is used to estimate $x(2)$ for various choices of stepsize. The following results are obtained.

Number of steps	approximate x(2)
1	-0.1881510417
2	-0.1131418135
4	-0.1037212734
8	-0.1029488697
16	-0.1028930652
32	-0.1028893052
64	-0.1028890611

i. Use these results to estimate $x(2)$ accurate to 5 decimal places.

ii. Estimate the errors in the approximation obtained using 4 steps and 8 steps.

iii. Hence calculate the effective order of the method at stepsize $h = 0.125$.

iv. Which numerical method do you think might have been used to get these results? Give a reason for your answer.

3. This question is about the differential equation

$$\frac{dy}{dt} = t\sqrt{y}.$$

(a) Use the Existence and Uniqueness Theorems to show that there is a unique solution satisfying the differential equation and the initial condition $y(0) = 3$. You do not need to find this solution.

(b) Use substitution to show that both $y_1(t) = 0$ and $y_2(t) = (t^2-1)^2/16$ satisfy the differential equation and the initial condition $y(1) = 0$.

(c) In (b) you showed that a certain initial value problem has two solutions. Why does this not contradict the Uniqueness Theorem?

4. Beth initially deposits \$400 in a savings account that pays interest at the rate of 3% per year compounded continuously. She also arranges for \$10 per week to be deposited automatically into the account.

(a) Assume that weekly deposits are close enough to continuous deposits so that we can reasonably approximate her account balance using a differential equation. Write an initial-value problem for her balance over time.

(b) Approximate Beth's balance after 4 years by solving the initial-value problem in (a).

(c) Use either *analyzer* or *dfield* from Matlab to work out how long will it be until Beth has \$1000 in her account. Explain clearly how you got your answer and include printouts of any Matlab output that you used to work out your answer.