DEPARTMENT OF MATHEMATICS

MATHS 260 Exercises on eigenvalues and eigenvectors

- The following exercises test your ability to find eigenvalues and eigenvectors.
- You should be able to do all of these exercises. If you cannot, you will need to revise how to find eigenvalues and eigenvectors *before* Lecture 17, when we will begin to use them extensively.
- The textbook explains how to find eigenvalues and eigenvectors in Chapter 3, pages 262–267 (page numbers correct for the 3rd edition).

For the following matrices, find the eigenvalues and eigenvectors

1. $\begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix}$
2. $\begin{pmatrix} -1 & 0 \\ 0 & 5 \end{pmatrix}$
3. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
4. $\begin{pmatrix} 3 & 2 \\ 0 & -2 \end{pmatrix}$
5. $\begin{pmatrix} -1 & 6 \\ 0 & 5 \end{pmatrix}$
6. $\begin{pmatrix} -5 & -2 \\ -1 & -4 \end{pmatrix}$
7. $\begin{pmatrix} 3 & 4 \\ 1 & 0 \end{pmatrix}$
8. $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$
9. $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$
10. $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$
11. $\begin{pmatrix} 3 & 0 & 0 \\ -2 & 7 & 0 \\ 4 & 8 & 1 \end{pmatrix}$

Answers

1. Eigenvalues 3 and -2 with eigenvectors $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$.
2. Eigenvalues -1 and 5 with eigenvectors $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$.
3. Eigenvalue 1 (repeated) with eigenvectors $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$.
4. Eigenvalues -2 and 3 with eigenvectors $\begin{pmatrix}-2\\5\end{pmatrix}$ and $\begin{pmatrix}1\\0\end{pmatrix}$.
5. Eigenvalues -1 and 5 with eigenvectors $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}1\\1\end{pmatrix}$.
6. Eigenvalues -3 and -6 with eigenvectors $\begin{pmatrix}1\\-1\end{pmatrix}$ and $\begin{pmatrix}2\\1\end{pmatrix}$.
7. Eigenvalues -1 and 4 with eigenvectors $\begin{pmatrix}1\\-1\end{pmatrix}$ and $\begin{pmatrix}4\\1\end{pmatrix}$.
8. Eigenvalues $\frac{3+\sqrt{5}}{2}$ and $\frac{3-\sqrt{5}}{2}$ with eigenvectors $\begin{pmatrix}2\\\sqrt{5}-1\end{pmatrix}$ and $\begin{pmatrix}-2\\\sqrt{5}+1\end{pmatrix}$.
9. Eigenvalue 1 (repeated) with eigenvector $\begin{pmatrix}1\\0\end{pmatrix}$ (there is only a single eigenvector in this case).
10. Eigenvalues 2, 1 and -1 with eigenvectors $\begin{pmatrix}1\\0\\0\end{pmatrix}$, $\begin{pmatrix}0\\1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\0\\1\end{pmatrix}$
11. Eigenvalues 3, 7 and 1 with eigenvectors $\begin{pmatrix}2\\1\\8\end{pmatrix}$, $\begin{pmatrix}0\\3\\4\end{pmatrix}$ and $\begin{pmatrix}0\\0\\1\end{pmatrix}$.