Department of Mathematics

Maths 260 Assignment 2

August 20, 2009 Due: 4pm, Tuesday, August 26, 2009

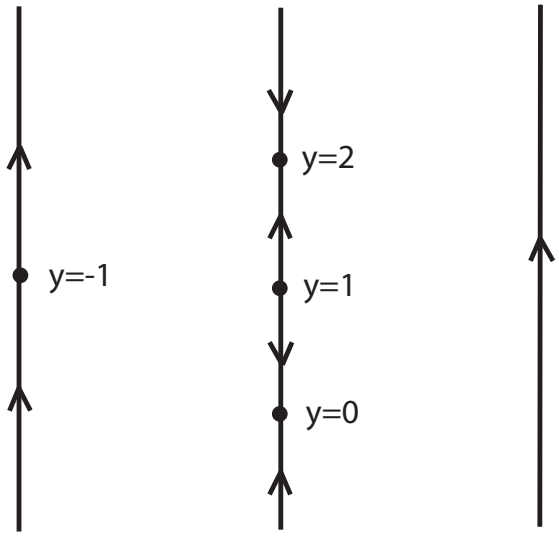
- Put your completed assignment in the appropriate box in the basement of the Maths/Physics building **before** 4pm on the date due.
- Late assignments or assignments placed in the wrong box will not be marked.
- Your assignment must be accompanied by a blue Mathematics Department coversheet. Copies of the coversheet are available from a box in the basement.

1. For each of the phase lines shown below:

(a) write down a differential equation that would have the phase line shown;

(b) describe the long term behaviour of solutions shown in the phase line;

(c) confirm that your answers to (a) and (b) are correct by using dfield to plot solutions to the differential equation you wrote down in (a). Hand in your dfield plots with your answers to this question.



2. This question is about the differential equation

$$\frac{dy}{dt} = \frac{y}{t} - 1, \quad t > 0.$$

(a) Find a one-parameter family of solutions to the differential equation.

(b) Use *analyzer* to plot the graphs of three different solutions to the differential equation. Draw all three graphs on the same picture, print your picture, and hand it in with your assignment.

(c) Describe the long term behaviour of all solutions to the differential equation.

3. This question is about the one-parameter family of differential equations

$$\frac{dy}{dt} = y^2 + 2y - a.$$

(a) For the case $a = 0$, find all equilibrium solutions and determine their type (e.g., sink, source). Sketch the phase line.

(b) Repeat (a) for the case $a = -4$.

(c) Now let a vary.

i. Locate the equilibrium solutions and determine their type for all values of a including any bifurcation values.

ii. Sketch the bifurcation diagram. Be sure to label the main features of the bifurcation diagram.

4. The following differential equation has been suggested as a model of the growth of a population of cockroaches in one of the campus restaurants:

$$\frac{dc}{dt} = kc(100 - c) - s.$$

In this equation, c represents the number of cockroaches, measured in thousands (so $c = 1$ means there are 1000 cockroaches in the restaurant) and t is time measured in weeks. The constants $k > 0$ and $s \geq 0$ are parameters in the model.

(a) Briefly say what each term in the model might represent physically, i.e., say what physical phenomenon is being modelled by each term.

(b) If c is very small ($c << 100$) and $s = 0$ then the population of cockroaches is observed to grow at a rate of about 10% per week. What value of k should be used in the model to match this observation?

(c) What is the maximum possible equilibrium population of cockroaches, according to this model?

5. (Challenge question) You can use Matlab to work out the answers to this question. Include printouts from Matlab to support any claims you make that are based on what you see with Matlab.

Consider the differential equation

$$\frac{dy}{dt} = k + 3y - y^3,$$

for k a constant.

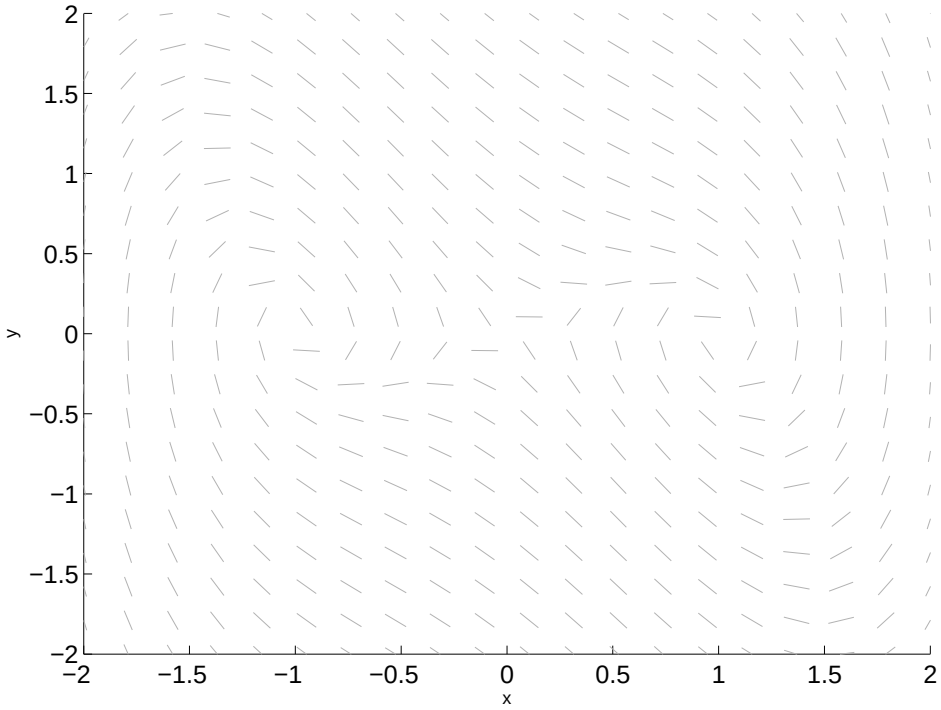
(a) Find two bifurcation values of k accurate to two significant figures. Describe how the behaviour of solutions changes as k passes through each bifurcation value.

(b) Sketch the bifurcation diagram.

6. The following two pictures show direction fields for the two sets of differential equations given. On each picture, sketch a representative sample of solution curves, making sure to show the direction of time with arrows. (Hand in this page with your solutions).

(a)

$$\dot{x} = y$$
$$\dot{y} = x - y - x^3$$



(b)

$$\dot{x} = y - x^2$$
$$\dot{y} = x$$