Some Personal Recollections of Oved Shisha

I first met Oved Shisha in 1982, at the Technion in Haifa, Israel, where he was spending a sabbatical year and I had a postdoctoral fellowship. Shisha was teaching a graduate course in approximation theory, and this was exceptionally lucky for me: my Ph.D. was on Padé approximation, and the narrow focus of my studies meant that I had graduated without knowing about Jackson's theorem. Shisha's courses were taught in the afternoons in his office in the prefabricated huts close to the Technion's Churchill building. The students would read sections of the textbooks of Cheney and Lorentz and then discuss them in class. Oved's taste for simplicity in mathematics frequently emerged in these discussions.

Sometimes the four or five participants would first have tea from the small urn down the passage and Oved would sympathetically listen to their hopes and aspirations and also to their personal problems. Since I had more free time than the graduate students, I was able to talk to him at greater length, and absorbed some of what he thought important in approximation theory, in current ideas and active areas of research. Our discussions led to one joint paper, on best rational approximation on the whole plane, partially a mixture of my interest in rational approximation and his interest in the existence and uniqueness theory of best approximation [83] (see "Publications of Oved Shisha," which follows).

From time to time Oved would become extremely busy with the latest issue of the *Journal of Approximation Theory*. He would spend hours in his office poring over galley proofs, checking language, style, and mathematical detail. After a marathon two- or three-day effort, he would emerge with a large sack of manuscripts over his shoulder and relieve himself of the burden until the next issue. The Journal was a very large part of his life, and he recalled on more than one occasion his joy in founding and successfully running it. He would also happily recall his days at Wright–Patterson Air Force Base, where he could arrange conferences on inequalities and invite many visitors, while enjoying generous support for research, and where the idea for the Journal apparently began.

Paul Erdős paid one of his frequent visits to the Technion in 1983, and Oved introduced me to him. The visit coincided with Paul's 70th birthday, and he gave one of his famous problem talks. Oved, Paul, Allan Pinkus, and the complex function theorist J. Milne Anderson began a paper on moment problems during this visit [93]. I carried Paul Erdős' suits to the local dry cleaner, and as a reward he gave me some of the best advice of

my career, namely to redirect my interest in quadrature theory by reading Freud's book on orthogonal polynomials. At a lunch after the talk, Paul Erdős, Oved Shisha, J. Milne Anderson, and the number theorist Emil Grosswald (then also on sabbatical at the Technion) discussed the issues of the day with their Technion colleagues.

Perhaps Oved's most lasting interest was the theory of integration. Over a period of 30 years, he investigated convergence of sequences of quadrature rules for unbounded functions, characterizing orders of growth of unbounded functions in terms of their Riemann sums, comparing the generalized Riemann integral and the Lebesgue integral, and relating convergence of Riemann sums to the prime number theorem [17, 46, 47, 64–66, 69, 72, 75, 76, 84, 87, 97, 101, 108]. It is perhaps fitting that his last published paper [111] should be on this topic. In it, he compared the generalized Riemann integral favorably with the Lebesgue integral, displaying some of its advantages. This is a controversial point of view, but in a way is characteristic of his approach to mathematics. He was essentially interested in foundations, and was not afraid to take a fresh look at well-established definitions and to examine them from a new perspective.

Another of his interests was monotone approximation: What is the degree of approximation to monotone functions by monotone polynomials of degree n? In [16] he proved a very simple Jackson theorem: if $f: [-1, 1] \to \mathbb{R}$, and $f' \geq 0$ in $[-1, 1]$, then for every $n \in \mathbb{N}$ there exists a polynomial P_n of degree at most n, such that $P_n' \geq 0$ in $[-1, 1]$, and

$$\|f - P_n\|_{L^\infty([-1, 1])} \leq C\omega\left(f', \frac{1}{n}\right),$$

where C is an absolute constant independent of f and n, and, in fact, is given explicitly. Here $\omega(f')$ denotes the usual first order modulus of continuity of f'. This started a whole new direction in quantitative approximation, one that is still receiving attention today, with some extremely difficult problems involving different phenomena for different orders of moduli of continuity. At a recent conference in Budapest (AFS95, August 1995), honoring Károly Tandori's 70th birthday, Dany Leviatan gave a survey talk on this topic. He mentioned the influence of Oved Shisha' work and that the methods he used can be used to prove best possible results in one direction. Oved would be happy with this; he delighted in simple mathematics based on good ideas. He always emphasized concise and clear presentation as the key to attracting researchers to the areas that he helped to popularize.

Oved had a long lasting interest in positive linear operators. Two of his favorite papers were those he wrote with Bert Mond in which they equipped the Bohman–Korovkin theorem with rates, transforming it from

a qualitative to a quantitative result [34, 35]. During one of our discussions, he modestly dismissed the originality of their contribution, saying what a treasure Bohman and Korovkin had left buried in the garden when they proved their theorem.

Toward the end of his sabbatical, Oved's father passed away and he moved temporarily to his parents' flat in Tel Aviv to take care of his father's concerns and his mother. On one of my visits we discussed his father's life, his Hungarian education and pride in Oved's success. He had some of Oved's papers bound into slim leather volumes. On another visit I recall him reading an AMS Carus Mathematical Monograph on complex analysis, looking for inspiration in the simplicity and elegance of the little book. He returned to the United States in 1982 after his sabbatical, but a few months later returned to Israel to settle his mother into a managed care home.

I met Oved a few times after that, at the University of Rhode Island in 1984, and at a Southeast Approximators' Conference in Auburn, Alabama, in 1986, where he gave an impressive invited talk on his contributions to Riemann integration. We talked often, but our mathematical discussions did not lead to any more papers.

Oved Shisha's legacy is this Journal which he started and in which he took so much pride.

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