MATHS 255FS Class Test: Overview of questions. Detach for reference

1. For any integer n, let $A(n)$ be the statement:

If n is odd, then $n = 5q + 1$ or $n = 5q + 3$ for some integer q.

(a) (**3 marks**) Write down the negation of $A(n)$.

(b) (**2 marks**) Write down the contrapositive of $A(n)$.

(c) (**2 marks**) Write down the converse of $A(n)$.

(d) (**3 marks**) Determine with reason whether the converse of $A(n)$ is true.

2. Let $A = \{x \in \mathbb{R} : x \neq 5\}$, $B = \{x \in \mathbb{R} : x \neq 4\}$ and define $f : A \to B$ by $f(x) = \frac{4x}{x-5}$.

(a) (**7 marks**) Show that f is one-to-one and onto.

(b) (**3 marks**) Determine the inverse f^{-1} of f.

3. Let $S = \{-7, -6, -4, -2, 0, 1, 2, 4, 5, 7\}$ be a subset of $\mathbb{Z}$, and let $\sim$ be a relation defined on S by $x \sim y$ if $4 \mid (x + 3y)$.

(a) (**6 marks**) Show that $\sim$ is an equivalence relation.

(b) (**4 marks**) Find all distinct equivalence classes.

4. (a) (**7 marks**) Proof by induction that for any $n \in \mathbb{N}$,

$$1 + \frac{1}{4} + \frac{1}{9} + \cdots + \frac{1}{n^2} \leq 2 - \frac{1}{n}.$$

(b) (**3 marks**) Let $A = \{1, 2, 3, 4\}$ and let $R = \{(1, 1), (1, 2), (2, 2), (2, 1)\}$ be a relation on A. Show that there exists no partial order Q on A such that $R \subseteq Q$.