1. (a) Suppose $\epsilon > 0$. If $|x| < 1$, then $|x^2| = |x|^2 \leq |x|$ and $|x^3| \leq |x|$. Let $\delta = \min\{1, \epsilon\}$. If $x \geq 0$, then

$$|f(x) - f(0)| = |x^2| \leq |x| < \epsilon \qquad \text{whenever } |x| < \delta.$$

If $x < 0$, then

$$|f(x) - f(0)| = |x^3| \leq |x| < \epsilon \qquad \text{whenever } |x| < \delta.$$

It follows that $f(x)$ is continuous at 0.

(b) Take $\epsilon = 1$ and let $\delta_1 > 0$ be a real number such that

$$|(x+2) - 3| < \epsilon \qquad \text{whenever } |x - 1| < \delta_1.$$

Then for any x with $0 < x - 1 < \delta_1$,

$$|f(x) - f(1)| = |(x+2) - 3 + 3 - f(1)| \geq ||x-1| - 2| \geq 2 - |x-1| \geq 2 - \epsilon = \epsilon.$$

It follows that $f(x)$ is not continuous at 1.

2. f is continous at 0 $\iff$ $(\forall\ \epsilon > 0)(\exists\ \delta_1 > 0)(\forall\ |y - 0| < \delta_1)$

$$|f(y) - f(0)| < \epsilon.$$

Suppose $|x| < \delta_2 := \sqrt{\delta_1/3}$, so that $|3x^2| < \delta_1$.
Let $\delta := \min\{\delta_1, \delta_2\}$. If $|x| < \delta$, then $|3x^2 - 0| < \delta_1$ and so

$$|f(3x^2) - f(0)| < \epsilon.$$

3. (a) Let $a \in \mathbb{R}$ and $b = f(a)$. f is continous at b $\iff$ $(\forall\ \epsilon > 0)(\exists\ \delta_1 > 0)(\forall\ |y - b| < \delta_1)(|f(y) - f(b)| < \epsilon)$.
Similarly, since $\delta_1 > 0$, g is continous at a $\implies$ $(\exists\ \delta > 0)(\forall\ |x - a| < \delta)(|g(x) - g(a)| < \delta_1)$.
Thus if $|x - a| < \delta$, then $|y - b| < \delta_1$ with $y = f(x)$ and so

$$|f(y) - f(b)| = |f(g(x)) - f(g(a))| < \epsilon.$$

(b) For $\epsilon > 0$, suppose $|x - 1| < 1$. Then

$$|f(x) - f(1)| = |x^3 - 1| = |(x-1)(x^2 + x + 1)| \leq |x - 1||x^2 + |x| + 1|.$$

Now $|x| \leq |x - 1| + 1 < 2$, so

$$|f(x) - f(1)| = |x^3 - 1| < |x - 1||4 + 2 + 1| = 7|x - 1|.$$

Suppose $|x - 1| < \delta_2 := \frac{\epsilon}{7}$ and set $\delta := \min\{1, \delta_2\}$. If $|x - 1| < \delta$, then

$$|f(x) - f(1)| = |x^3 - 1| < 7|x - 1| < 7\delta_2 = \epsilon.$$

Thus f is continous at 1.

Note that $f(-1) = -1$. Take $\epsilon = 1$ and let $\delta_1 > 0$ be a real number such that

$$|3x + 3| < \epsilon \qquad \text{whenever } |x + 1| < \delta_1.$$

Then for any x with $0 < x + 1 < \delta_1$,

$$|f(x) - f(-1)| = |3x + 1| = |3x + 3 - 2| \geq ||3x + 3| - 2| \geq 2 - |3x + 3| \geq 2 - \epsilon = \epsilon.$$

It follows that $f(x)$ is not continuous at -1.