1. (**Final Exam 2004FS**) Let f be a function from $\mathbb{R}$ to itself defined by

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 0, \\ x^3 & \text{if } 0 < x \leq 1, \\ x + 2 & \text{if } 1 < x. \end{cases}$$

(a) Prove from the first principles that $f(x)$ is continuous at 0.

(b) Prove from the first principles that $f(x)$ is *not* continuous at 1.

2. Let $f(x)$ be a continous function from $\mathbb{R}$ to itself. Proof from the first principles that $g(x) := f(3x^2)$ is continous at $x = 0$.

3. (**Final Exam 2004SS**)

(a) Use first principles to show that if $f, g : \mathbb{R} \to \mathbb{R}$ are continous, then $f \circ g$ is continous.

(b) Use first principles to show that $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} x^3 & \text{if } x \geq -1, \\ 3x & \text{if } x < -1 \end{cases}$$

is continuous at 1, but not at -1.