1. (**Finite subgroup test**) Let $(G, *)$ be a group and H a non-empty finite subset of G. Suppose $x * y \in H$ for any $x, y \in H$.

(a) For any $x \in H$, there are positive integers m, n such that $n \neq m$ and $x^n = x^m$.

(b) Show that $H \leq G$.

2. Let G be a group.

(a) Suppose L and K are subgroup of G. Show that $L \cap K$ is a subgroup of G.

(b) Let $L \leq G$ and $K \leq L$. Show that $K \leq G$.

(c) Let H be a subgroup of G and $x \in G$. Show that the left coset $x * H$ is a subgroup of G if and only if $x * H = H$.

3. Let $\mathrm{GL}_2(\mathbb{R})$ be all invertible 2×2 real matrices and det the determinant of matrix.

(a) Show that $\mathrm{GL}_2(\mathbb{R})$ is a group under matrix multiplication.

(b) Show det is a group homomorphism from $\mathrm{GL}_2(\mathbb{R})$ to R^*, where $R^* = \mathbb{R} \setminus \{0\}$ is the group under multiplication.

(c) Show det is not a group isomorphism.

(d) Let $\mathrm{SL}_2(\mathbb{R}) = \{X \in \mathrm{GL}_2(\mathbb{R}) : \det(X) = 1\}$. Show that $\mathrm{SL}_2(\mathbb{R}) \leq \mathrm{GL}_2(\mathbb{R})$.