1. Let A and B be finite nonempty sets such that $|A| = |B| < \infty$, and let $f : A \to B$ be a function. Show that f is one-to-one if and only if f is onto.

2. Let $A = \{x \in \mathbb{R} : x \neq 1\}$, $B = \{x \in \mathbb{R} : x \neq 2\}$ and define $f : A \to B$ by $f(x) = \frac{2x}{x-1}$.

(a) Show that f is one-to-one and onto.

(b) Determine the inverse f^{-1} of f.

(c) Determine $f \circ f^{-1} \circ f$.

3. Let $f : A \to B$ and $g : B \to C$ be functions. Show that if $g \circ f$ is onto and g is one-to-one then f is onto.

4. Let $f : A \to B$ be a function. Define a new function $F : \mathcal{P}(A) \to \mathcal{P}(B)$ by declaring that, for $S \subseteq A$,

$$F(S) = \{\, f(a) : a \in S \,\}.$$

(a) Show that F is one-to-one if and only if f is one-to-one.

(b) Show that F is onto if and only if f is onto.

5. Let $A = \{1 - \frac{1}{2^n} : n \in \mathbb{N}\}$ and view A as a totally ordered set under the usual ordering on $\mathbb{R}$. Show $A \simeq \mathbb{N}$ as posets.