1. Prove by mathematical induction that, for every $n \in \mathbb{N}$, $n^5 - n$ is divisible by 5.

2. A sequence of integers is defined recursively by $x_1 = 1$, $x_2 = 4$ and $x_n = 2x_{n-1} - x_{n-2} + 2$ for $n \geq 3$. Conjecture a formula for x_n and prove it using mathematical induction.

3. Let U be a set and let $A_1, A_2, \ldots, A_n$ be n subsets of U, where $n \geq 2$. Show that

$$(A_1 \cap A_2 \cap \ldots \cap A_n)_U^C = (A_1)_U^C \cup (A_2)_U^C \cup \ldots \cup (A_n)_U^C.$$

4. Use mathematical induction to show that for every real number $x > -1$ and every integer n,

$$(1 + x)^n \geq 1 + nx.$$