1. Let A, B and C be statements. Construct truth tables for the following statements. For each statement, state whether it is a tautology, a contradiction or neither.

(a) $(A \iff B) \iff ((A \implies B) \wedge (B \implies A))$.

(b) $(A \implies (B \vee C)) \iff ((A \vee B) \implies C)$.

2. Suppose n is an integer. Let $A(n)$ be the statement that if $n = 4q + 1$ or $n = 4q + 3$ for some integer q, then n^2 is odd.

(a) Translate $A(n)$ into symbols.

(b) Write down the negation of $A(n)$.

(c) Write down the converse of $A(n)$.

(d) Write down the contrapositive of $A(n)$.

(e) Use a direct proof to show that $(\forall n \in \mathbb{Z})\ A(n)$.

(f) Use a proof by contradiction to show that the converse of $A(n)$ is true for all $n \in \mathbb{Z}$.

(g) Determine whether or not the contrapositive of $A(n)$ is true.

3. Translate the following statements into symbols and disprove each of them.

(a) For any integers $x, y, z \in \mathbb{Z}$, if $x + z$ is odd, then $x + y$ and $y + z$ are both odd.

(b) There is an integer $n \in \mathbb{Z}$ such that $n^3 + n + 1$ is even.

(c) For every two positive integers $a, c \in \mathbb{Z}$, there exists a positive integer b such that $a + b = c$.