1. For $\epsilon > 0$, let $N \in \mathbb{N}$ with $N > \frac{17-18\epsilon}{4\epsilon}$. If $n > N$, then

$$\left|\frac{3n+5}{2n+9} - \frac{3}{2}\right| = \left|\frac{-17}{4n+18}\right| \leq \frac{17}{4N+18} \leq \frac{17}{4\frac{17-18\epsilon}{4\epsilon}+18} = \epsilon.$$

Thus $\lim_{n\to\infty} \frac{3n+5}{2n+9} = \frac{3}{2}$.

2. Since $\lim_{n\to\infty} a_n = a$, it follows that for $\epsilon > 0$, there exists $N_1 \in \mathbb{N}$ such that for all $n > N_1$,

$$|a_n - a| < \epsilon.$$

Similarly, since $\lim_{n\to\infty} b_n = a$, there exists $N_2 \in \mathbb{N}$ such that for all $n > N_2$,

$$|b_n - a| < \epsilon.$$

Let $N = \max\{N_1, N_2\}$ and suppose $m > N$.
If $m = 2k-1$ for some $k \in \mathbb{N}$, then $c_m = a_m$ and $m > N \geq N_1$, so that

$$|c_m - a| = |a_m - a| < \epsilon.$$

If $m = 2k$ for some $k \in \mathbb{N}$, then $c_m = b_m$ and $m > N \geq N_2$, so that

$$|c_m - a| = |b_m - a| < \epsilon.$$

It follows that for all $m > N$,

$$|c_m - a| < \epsilon,$$

so that $\lim_{m\to\infty} c_m = a$.

3. (a) $a_{n+1} = \dfrac{(n+1)^2}{2^{n+1}}$ and

$$a_{n+1} - a_n = \frac{(n+1)^2}{2^{n+1}} - \frac{n^2}{2^n} = \frac{(n+1)^2 - 2n^2}{2^{n+1}} = \frac{n(2-n)+1}{2^{n+1}} < \frac{n(2-4)+1}{2^{n+1}} < 0$$

since $n > 3$. Thus (a_n) is monotonic decreasing when $n > 3$.

(b) $a_1 = 1/2$, $a_2 = 1$, $a_3 = 9/8$, and $a_n \leq a_4 = 1$ for $n > 4$, since a_n is decreasing when $n \geq 4$. Thus

$$0 < a_n \leq \frac{9}{8}$$

for all $n \in \mathbb{N}$.

(c) $2^n = (1+1)^n = 1 + n + \frac{n(n-1)}{2} + \frac{n(n-1)(n-2)}{6} + \cdots + 1 > \frac{n(n-1)(n-2)}{6}$, so

$$0 \leq a_n < \frac{n^2}{\frac{n(n-1)(n-2)}{6}} = \frac{6n^2}{n(n-1)(n-2)} = \frac{6}{(1-1/n)(n-2)}$$

Thus lub $\{a_n : n \in \mathbb{N}\} = 9/8 \in \{a_n : n \in \mathbb{N}\}$ and glb $\{a_n : n \in \mathbb{N}\} = 0 \notin \{a_n : n \in \mathbb{N}\}$.

(d) $\lim_{n\to\infty} a_n = \text{glb}\ \{a_n : n \in \mathbb{N}\} = 0$.