1. Let $f(x), g(x)$ and $h(x)$ be polynomials in $\mathbb{Z}_7[x]$ defined by

$$f(x) = x^5 + 2x^2 + x + 4, \qquad g(x) = 3x^2 + 2.$$

Here for simple, we denote $\bar{a}$ by a for $\bar{a} \in \mathbb{Z}_7$.

(a) Find quotient $q(x)$ and remainder $r(x)$ when $f(x)$ is divided by $g(x)$.

(b) Find the monic greatest common divisor $d(x)$ of $f(x)$ and $g(x)$, and find polynomials $u(x)$ and $v(x)$ such that

$$d(x) = f(x)u(x) + g(x)v(x).$$

2. Let $A = \mathbb{R}\backslash\{-1\}$ and let $*$ be an operation on A defined by $a * b = a + b + ab$.

(a) Check $(1 + a)(1 + b) = 1 + a * b$ for all $a, b \in A$. Hence show that $*$ is an associative binary operation on A.

(b) Show $(A, *)$ is an abelian group.

3. Let G be a group with identity e and let $a \in G$.

(a) Show that $e^{-1} = e$.

(b) Let $a^n = a \cdot a \cdots a$ (n terms). If $x^n = e$ with $n \geq 2$, then show that $x^{-1} = x^{n-1}$.