

1. The algorithm gives us

n	x	y	
1173	1	0	r_1
957	0	1	r_2
216	1	-1	$r_3 = r_1 - r_2$
93	-4	5	$r_4 = r_2 - 2r_3$
30	9	-11	$r_5 = r_3 - 2r_4$
3	-31	38	$r_5 = r_3 - 3r_4$
0	319	-391	$r_5 = r_3 - 10r_4$

from which we see that $\gcd(1173, 957) = 3$ and $3 = 1173 \cdot (-31) + 957 \cdot 38$.

2. Suppose that $a, b, c, x, y \in \mathbb{Z}$ with $c \mid a$ and $c \mid b$. Then there exist $p, q \in \mathbb{Z}$ with $a = cp$ and $b = cq$. But then

$$ax + by = (cp)x + (cq)y = c(px + qy),$$

and $(px + qy) \in \mathbb{Z}$, so $c \mid (ax + by)$ as required.

3. (a) Since $m \mid (42n + 17)$ and $m \mid (7n + 2)$, it follows that

$$m \mid 5 = (42n + 17) + (-6)(7n + 2).$$

But 5 is a prime, so $m = 1$ or 5. Now $m > 1 \implies m = 5$.

(b) Let $d = \gcd(3n + 2, 2n + 1)$. Since $3n + 2 = (2n + 1) \cdot 1 + (n + 1)$, it follows that

$$d = \gcd(2n + 1, n + 1).$$

But $1 = 2(n + 1) + (-1)(2n + 1)$, so $d \mid 1$ and $d = 1$.