1. Indicate whether each of the following relations on the given set is reflexive, symmetric, antisymmetric, or transitive. Explain your answers.

(a) $A = \mathcal{P}(\{1, 2, \cdots, 10\})$ and let the relation $\sim$ on A be defined as follows: for all $X, Y \in A$, $X \sim Y$ iff for all $x \in X$, there exists $y \in Y$ such that $y|x$.

(b) $B = \{x \in \mathbb{Z} : x = 3t, \exists\, t \in \mathbb{Z}\} = 3\mathbb{Z}$ and $x\rho y$ iff $(\exists\, w \in B)(xw = y)$.

(c) $C = \{x \in \mathbb{N} : x = 2t, \exists\, t \in \mathbb{N}\} = 2\mathbb{N}$ and $x\rho y$ iff $x + y = 6$.

(d) $D = \{x \in \mathbb{N} : x = 4t, \exists\, t \in \mathbb{N}\} = 4\mathbb{N}$ and $x\rho y$ iff $x + 2y = 6$.

2. Let S be the set $\{1, 2, 3, 4, 5, 6\}$.

(a) Let ρ be the relation

$$\rho = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6), (1,2), (2,1), (1,3), (3,1), (2,3), (3,2), (4,5), (5,4)\}$$

Verify that ρ is an equivalence relation. Find all equivalence classes and check the collection of distinct classes is a partition of S.

(b) Let $S_1 = \{1, 2\}$, $S_2 = \{3, 4, 6\}$ and $S_3 = \{5\}$. Verify that $\{S_1, S_2, S_3\}$ is a partition of S. Define an equivalence relation ρ such that each S_i is an ρ-equivalence class.

3. Let $A = \mathbb{Z}$ and $\sim$ be a relation on A such that $x \sim y$ iff $3|x + 2y$.

(a) Show $\sim$ is an equivalence relation.

(b) Show that $[0], [1], [2]$ are all distinct equivalence classes.