THE UNIVERSITY OF AUCKLAND **MATHS 255**

TERM TEST, Second Semester 2004

Principles of Mathematics

(Time allowed: 50 MINUTES)

Family Name _____

Given Names _____

By signing this cover sheet I confirm that I am the student whose name appears above:

Signed _____

- Answer all the questions in the spaces provided.
- The marks for each part of each question are shown.
- The total number of marks is 40.

Family Name _____

Given Names _____

ID Number _____

Question	Mark
1	
2	
3	
4	
5	
Total	

1. (a) Complete the truth table for the following proposition P

A	B	$A \vee B$	$\Longrightarrow$	$\neg (A \Longrightarrow B)$
T	T			
T	F			
F	T			
F	F			

(2 marks)

(b) Is this a tautology? Explain. (1 mark)

(c) Write the contrapositive of P (1 mark)

(d) Express $\neg(A \implies B)$ using $\neg$ and $\wedge$ only. (2 marks)

(e) Write the converse of P using the expression in (d). (1 mark)

(f) Is the converse of P true? Give a reason. (1 mark)

2. (a) Give a direct proof that if n is an even integer then $5n + 3$ is odd. (4 marks)

(b) Use proof by contradiction to show that 48 cannot be written as the sum of three integers, an odd number of which are odd. (4 marks)

NOTE: The proof consists of two simple cases.

3. (a) Show that if $A, B, C \subseteq U$ then $B \subseteq C \implies A \setminus C \subseteq A \setminus B$. (4 marks)

(b) Let $f : Q \to R$ and $g : P \to Q$ be functions such that $f \circ g$ is onto.
Show that if g is not onto then f cannot be one to one. (5 marks)

4. Prove by induction that for every $n \in \mathbb{N}$, $81 \mid (10^{n+1} - 9n - 10)$. (7 marks)

5. Consider the poset $(S, \preceq)$ where S is the set of points in $\mathbb{R}^2$:

$$S = \{(2,5),(5,2),(3,5),(5,6),(6,2),(6,5),(6,6)\} \text{ and}$$
$$(a,b) \preceq (c,d) \iff (a \leq c) \wedge (b \leq d).$$

(a) Draw a lattice diagram for $(S, \preceq)$. (2 marks)

(b) Find any maximal, minimal, greatest and least elements. (2 marks)

(c) Find a subset containing $(3,5)$ which has no lower bound. (2 marks)

(d) Find a subset which is bounded below but has no greatest lower bound. (2 marks)