TERM TEST, First Semester 2004

1. Let $A(n)$ be the implication "if n is even then $n+1$ is prime".

(a) Write down the converse of $A(n)$. (2 marks)

(b) Write down the contrapositive of $A(n)$. (2 marks)

(c) Write down the negation of $A(n)$. (2 marks)

In each case you should simplify as appropriate, for example changing "$n+1$ is not prime" to "$n+1$ is composite".

2. Let a and b be integers.

(a) Give a direct proof that if a is even or b is even then a^2b is even. (4 marks)

(b) Use proof by contradiction to show that if a^2b is even then a is even or b is even. (5 marks)

3. Let $f : \mathbb{R} \to \mathbb{R}$ be a function, and define a new function $g : \mathbb{R} \to \mathbb{R}$ by declaring that $g(x) = f(x+2)$, for $x \in \mathbb{R}$.

(a) Show that if f is one-to-one if and only if g is one-to-one. (7 marks)

(b) Show that if g is onto then f is onto. (4 marks)

4. Prove by induction that for every $n \in \mathbb{N}$, $n^2 + 3n + 1$ is odd. (7 marks)

5. Let $a, b \in \mathbb{Z}$. Put $X = \{ n \in \mathbb{Z} : n \mid a \wedge n \mid b \}$ and $Y = \{ n \in \mathbb{Z} : n \mid a \wedge n \mid a+b \}$. Show that $X = Y$. (7 marks)