1. (a) Complete the truth table for the following proposition P

A	B	$A \vee B$	$\implies$	$\neg$	$(A \implies B)$
T	T	T	F	F	T
T	F	T	T	T	F
F	T	T	F	F	T
F	F	F	T	F	T

(b) Is this a tautology? Explain.
It is NOT a tautology, as it includes F entries and so the implication is not always true.

(c) Write the contrapositive of P
$(A \implies B) \implies \neg(A \vee B)$

(d) Express $\neg(A \implies B)$ using $\neg$ and $\wedge$ only.
$A \wedge \neg B$

(e) Write the converse of P using the expression in (d).
$A \wedge \neg B \implies A \vee B$

(f) Is the converse of P true? Give a reason.
Yes it is true. $A \wedge \neg B \implies A \implies A \vee B$ If $A \wedge \neg B$ is true then A is true, so $A \vee B$ is true.

2. (a) Give a direct proof that if n is an even integer then $5n + 3$ is odd.
n is even $\iff n = 2pp \in \mathbb{Z}$ so $5n + 3 = 5(2p) + 3 = 2(5p + 1) + 1$ which is odd.

(b) Use proof by contradiction to show that 48 cannot be written as the sum of three integers, an odd number of which are odd.
NOTE: The proof consists of two simple cases.
Either we have one odd number and two even or three odd. Assume the result is true and get a contradiction. Case 1 one odd number $x = 2p + 1, y = 2q, z = 2r$ $48 = 2p + 1 + 2q + 2r = 2(p + q + r) + 1$ odd but $48 = 2 \cdot 24$ even contradiction. Case 2 three odd numbers $x = 2p+1, y = 2q+1, z = 2r+1$ $48 = 2p+1+2q+1+2r+1 = 2(p+q+r+1)+1$ odd but $48 = 2 \cdot 24$ even contradiction.

3. (a) Show that if $A, B, C \subseteq U$ then $B \subseteq C \implies A \setminus C \subseteq A \setminus B$.
$B \subseteq C \iff (x \in B \implies x \in C) \iff (x \notin C \implies x \notin B)$
$x \in A \setminus C \iff (x \in A) \wedge (x \notin C) \implies (x \in A) \wedge (x \notin B) \iff x \in A \setminus B$

(b) Let $f : Q \to R$ and $g : P \to Q$ be functions such that $f \circ g$ is onto.
Show that if g is not onto then f cannot be one to one.
Since g is not onto $\exists q \in Q : \nexists p \in P : g(p) = q$.
Now consider $f(q) \in R$. $f \circ g$ is onto, so $\exists p' \in P : (f \circ g)(p') = f(q)$.
$g(p') \neq q$ because there is no such $p \in P : g(p) = q$, but $f(g(p')) = f(q)$, so f not one to one.

4. Prove by induction that for every $n \in \mathbb{N}$, $81 \mid (10^{n+1} - 9n - 10)$.
$P(1)$ true. $10^2 - 9 - 10 = 100 - 19 = 81$ and $81 \mid 81$.
If $P(k)$ then $10^{k+1} - 9k - 10 = 81n$, so $10^{k+1} = 81n + 9k + 10$
Now consider

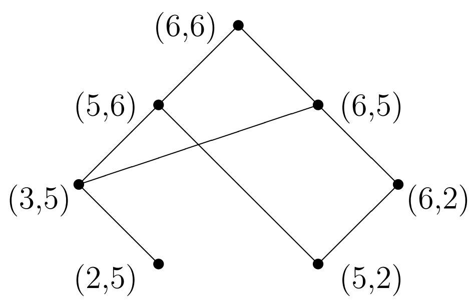
$$\begin{aligned} 10^{k+2} - 9(k+1) - 10 &= 10 \cdot 10^{k+1} - 9k - 10 \\ &= 10(81n + 9k + 10) - 9k - 19 \\ &= 81 \cdot 10n + 90k + 100 - 9k - 19 \\ &= 81 \cdot 10n + 81k + 81 \\ &= 81(10n + k + 1). \end{aligned}$$

So $P(k+1)$ also true so $P(n)$ true $\forall n \in \mathbb{N}$.

5. Consider the poset $(S, \preceq)$ where S is the set of points in $\mathbb{R}^2$:

$$S = \{(2,5), (5,2), (3,5), (5,6), (6,2), (6,5), (6,6)\} \text{ and}$$
$$(a,b) \preceq (c,d) \iff (a \leq c) \wedge (b \leq d).$$

(a) Draw a lattice diagram for $(S, \preceq)$.



(b) Find any maximal, minimal, greatest and least elements. $(6,6)$ is both maximal and greatest. $(2,5)$ and $(5,2)$ are minimal.

(c) Find a subset containing $(3,5)$ which has no lower bound. $\{(3,5),(6,2)\}$ or $\{(3,5),(5,2)\}$. There are others.

(d) Find a subset which is bounded below but has no greatest lower bound. $\{(5,6),(6,5)\}$