NB: Please deposit your solutions in the appropriate box **by 4 p.m. on the due date**. Late assignments or assignments placed into incorrect boxes will not be marked. Use a mathematics department cover sheet: these are available from outside the Resource Centre.

1. Find all solutions to the following Diophantine equations:

(a) $72x + 78y = 13$.

(b) $72x + 78y = 12$.

(c) $77x + 240y = 53$.

2. Solve the equation $\overline{53} = \overline{77} \cdot_{240} \overline{x}$ in $\mathbb{Z}_{240}$.

3. (a) Find all integers $x \in \mathbb{Z}$ such that

$$2x^2 - 3x \equiv 6 \pmod 7.$$

NOTE: The modulus is small enough that you can check each of the cases individually.

(b) Find all integers $x \in \mathbb{Z}$ such that

$$28x \equiv 42 \pmod{70}.$$

4. Use congruence to show that for every $n \in \mathbb{N}$, $(5n + 6)(13n + 7)(7n + 8)$ is divisible by 6.
HINT: Reduce each term to its simplest form modulo 6 allowing for minus terms if required.

5. (a) Use the Euclidean Algorithm for $\mathbb{R}[x]$ to find a greatest common divisor in $\mathbb{R}[x]$ of $f(x) = 2x^3 - 5x^2 - 9$ and $g(x) = x^3 - 7x - 6$.

(b) Find polynomials $u(x)$ and $v(x)$ such that

$$d(x) = f(x)u(x) + g(x)v(x).$$

NOTE: You will need to use a row method similar to that in assignment 3 q6.

6. Prove proposition 14 in the week 8 notes: Let $(G, *)$ be a group with identity element e.

(a) If $x \in G$ satisfies $x * x = x$, then $x = e$.

(b) If $x, y \in G$ satisfy $x * y = y$, then $x = e$.
[Put another way, if $x * y = y$ for *some* $y \in G$ then $x * y = y$ for *every* $y \in G$.]

7. Let $G = \{a, b, c, d, e\}$. Given that $*$ is a group operation on G, complete the following Cayley Table for $*$:

$*$	a	b	c	d	e
a					
b					d
c					
d					
e			e		b

8. Consider the following set K of matrices:

$$K = \left\{ \begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix} : x \in \mathbb{R} \right\}.$$

Prove that K is a group under matrix multiplication.