MATHS 255FS Class Test: Overview of questions. Detach for reference

1. For any integer n, let $A(n)$ be the statement:

If n is even, then n^2 is even and $n+1$ is odd.

(a) (**3 marks**) Write down the negation of $A(n)$.

(b) (**2 marks**) Write down the contrapositive of $A(n)$.

(c) (**2 marks**) Write down the converse of $A(n)$.

(d) (**3 marks**) Determine with reason whether it is true that $\forall\, n \in \mathbb{N}\ A(n)$.

2. (a) (**7 marks**) Proof by induction that $4^{3n} - 1$ is divisible by 9, for any nonnegative integer n.

(b) (**3 marks**) Use proof by contradiction to show that for any integers a, b if $a+b$ is odd, then a is odd or b is odd.

3. Let $S = \{-7, -6, -2, 0, 1, 4, 5, 7\}$ be a subset of $\mathbb{Z}$, and let $\sim$ be a relation defined on S by $x \sim y$ if $3 \mid (x + 2y)$.

(a) (**6 marks**) Show that $\sim$ is an equivalence relation.

(b) (**4 marks**) Find all distinct equivalence classes.

4. (a) (**6 marks**) Let the function $f : \mathbb{Z} \to \mathbb{Z}$ be defined by $f(x) = 3x + 5$. Show that f is one-to-one but not onto.

(b) (**4 marks**) Let g and h be bijective functions from $\mathbb{Z}$ to $\mathbb{Z}$. Show that $(g \circ h)^{-1} = h^{-1} \circ g^{-1}$.