**1.**

| $*$ | $R_0$ | $R_{120}$ | $R_{240}$ | $V$ | $D'$ | $D''$ |
|---|---|---|---|---|---|---|
| $R_0$ | $R_0$ | $R_{120}$ | $R_{240}$ | $V$ | $D'$ | $D''$ |
| $R_{120}$ | $R_{120}$ | $R_{240}$ | $R_0$ | $D'$ | $D''$ | $V$ |
| $R_{240}$ | $R_{240}$ | $R_0$ | $R_{120}$ | $D''$ | $V$ | $D'$ |
| $V$ | $V$ | $D''$ | $D'$ | $R_0$ | $R_{240}$ | $R_{120}$ |
| $D'$ | $D'$ | $V$ | $D''$ | $R_{120}$ | $R_0$ | $R_{240}$ |
| $D''$ | $D''$ | $D'$ | $V$ | $R_{240}$ | $R_{120}$ | $R_0$ |

**2.** (a) For all $a, b \in \mathbb{R}$

$$(1+a)(1+b) = 1 + a + b + ab = 1 + a * b.$$

If $*$ is not a binary operation on $A$, then $a * b \notin A$ for some $a, b \in A$, so that $a * b = -1$, since $a * b \in \mathbb{R}$ and $A = \mathbb{R}\backslash\{-1\}$. Thus $(1+a)(1+b) = 1 + a * b = 0$ and $a = -1$ or $b = -1$. Contradiction.

For $a, b, c \in A$, $((1+a)(1+b))(1+c) = 1+(a*b)*c$ and $(1+a)((1+b)(1+c)) = 1+a*(b*c)$. Since $((1+a)(1+b))(1+c) = (1+a)((1+b)(1+c))$ in $\mathbb{R}$, it follows that $(a*b)*c = a*(b*c)$.

(b) 0 is the identity of $(A, *)$, since $0 * a = 0 + a + 0a = a = a * 0$ for all $a \in A$.

For $a \in A$, if $b$ is its inverse, then $a * b = 0 \iff (1+a)(1+b) = 1 \iff b = \frac{-a}{1+a}$. Note that $b = \frac{-a}{1+a} \neq -1$, since $(1+a)(1+b) = 1$. Check $a * \frac{-a}{1+a} = \frac{-a}{1+a} * a = 0$, so $\frac{-a}{1+a}$ is the inverse of $a$.

Thus $(A, *)$ is a group.

(c) Since $0 \in B$, it follows that $B \neq \emptyset$.

For any $a, b \in B$, $a*b = ab+a+b \in \mathbb{Q}$. But $a*b \in A$, so $a*b \neq -1$ and $a*b \in B = \mathbb{Q}\backslash\{-1\}$. If $a \in B$, then its inverse in $A$ is $\frac{-a}{1+a}$, which is an element of $B = A \cap \mathbb{Q}$.

Thus $(B, *) \leq (A, *)$.

**3.** Since $L \leq H$, it follows that $L \neq \emptyset$.

For any $a, b \in L$, $ab^{-1} \in L$, since $L \leq H$. Thus $L \leq G$ by One-step test.

**4.** Suppose $\phi : \mathbb{R} \simeq \mathbb{Z}$. Then $\phi(0) = 0$ and $\phi(a) = 1$ for some $a \in \mathbb{R}$. If $\phi(\frac{a}{2}) = n \in \mathbb{Z}$, then $1 = \phi(a) = \phi(\frac{a}{2} + \frac{a}{2}) = \phi(\frac{a}{2}) + \phi(\frac{a}{2}) = 2n$ and $n = \frac{1}{2} \in \mathbb{Z}$. A contradiction.