1. (**Final exam, 2003SS**) Let $A = \{x \in \mathbb{Z} : -10 \leq x \leq 8\}$. Let $f : A \to A$ be defined as follows: For all $x \in A$, $f(x)$ is the remainder when x is divided by 4. [You are not asked to prove that f is a function.]

(a) (i) Find $f(7)$ and $f(-7)$.
  (ii) Determine whether or not f is one-to-one.
  (iii) Determine whether or not f is onto.

(b) Let $g : \mathcal{P}(A) \to \mathcal{P}(A)$ be defined as follows: For all $X \in \mathcal{P}(A), g(X) = \{a \in A : f(a) \in X\}$. [You are not asked to prove that g is a function.]
  (i) What is $g(\{-1, 0, 1\})$?
  (ii) Determine whether or not g is one-to-one.
  (iii) Determine whether or not g is onto.

(c) An equivalence relation is defined on A as follows: For all $a, b \in A$, $a \sim b$ if and only if $f(a) = f(b)$. [You are not asked to prove that $\sim$ is an equivalence relation.]
  (i) List all elements of the set $S = \{a \in A : a \sim 7\}$.
  (ii) Write down all of the equivalence classes under the relation $\sim$.

2. Let $f : A \to B$ be a function, $X, Y \in \mathcal{P}(A)$.

(a) Show that $f(X) \setminus f(Y) \subseteq f(X \setminus Y)$.

(b) Show that $f(X) \setminus f(Y) = f(X \setminus Y)$ for all $X, Y \in \mathcal{P}(A)$ if and only if f is one-to-one.

3. Let $A = \{\frac{1}{2n} : n \in \mathbb{Z}\backslash\{0\}\}$. Show that $(A, \leq) \simeq (\mathbb{Z}\backslash\{0\}, \leq)$ as posets.