

1. Prove by mathematical induction that, for every  $n \in \mathbb{N}$ ,  $12^n - 3^n$  is divisible by 9.
2. A sequence of integers is defined recursively by  $x_1 = 1$ ,  $x_2 = 2$  and  $x_n = x_{n-1} + 2x_{n-2}$  for  $n \geq 3$ . Conjecture a formula for  $x_n$  and prove it using mathematical induction.

3. Let  $U$  be a set and let  $A_1, A_2, \dots, A_n$  be  $n$  subsets of  $U$ , where  $n \geq 2$ . Show that

$$(A_1 \cap A_2 \cap \dots \cap A_n)^C = (A_1)^C \cup (A_2)^C \cup \dots \cup (A_n)^C.$$

4. Use mathematical induction to show that  $3^n > n^2$  for every  $n \in \mathbb{N}$ .