MATHS 255 RS

1. We have the truth table

A	B	$A \implies B$	$\sim A$	$\sim A \lor B$
Т	Т	Т	\mathbf{F}	Т
Т	\mathbf{F}	\mathbf{F}	\mathbf{F}	F
F	Т	Т	Т	Т
F	F	Т	Т	Т

Notice that the columns for $A \implies B$ and $\sim A \lor B$ are identical. Therefore $A \implies B$ and $\sim A \lor B$ are logically equivalent.

- **2.** (a) "x and y are both odd but xy is even."
 - (b) "If xy is odd then x and y are both odd."
 - (c) "If xy is even then either x is even or y is even."
 - (d) Suppose x and y are odd. Then x = 2k + 1 and y = 2m + 1 for some integers k, m. Thus xy = (2k+1)(2m+1) = 4km + 2k + 2m + 1 = 2(2km + m + k) + 1. Since 2km + m + k is an integer, it follows that xy is odd.
 - (e) Suppose, for a contradiction that x, y are both odd but xy is even. Since x and y are odd, x = 2k + 1 and y = 2m + 1 for some integers k, m, so that xy = (2k + 1)(2m + 1) = 4km + 2k + 2m + 1 = 2(2km + m + k) + 1. Thus xy is odd. A contradiction.
 - (f) The converse of P is the statement

"if xy is odd then x and y are both odd."

The converse is true. Suppose, for a contradiction that xy is odd and either x or y is even, say x is even. Then x = 2k for some integer k and so xy = 2ky is even, which is a contradiction.