1. For $\epsilon > 0$, let $N \in \mathbb{N}$ with $N > \frac{6-27\epsilon}{9\epsilon}$. If $n > N$, then

$$\left|\frac{n+5}{3n+9} - \frac{1}{3}\right| = \left|\frac{6}{9n+27}\right| \leq \left|\frac{6}{9N+27}\right| \leq \frac{6}{9\frac{6-27\epsilon}{9\epsilon} + 27} = \epsilon.$$

Thus $\lim_{n\to\infty} \frac{n+5}{3n+9} = \frac{1}{3}$.

2. Since $\lim_{n\to\infty} a_n = a$, it follows that for $\epsilon > 0$, there exists $N_1 > 0$ such that for all $n > N_1$,

$$|a_n - a| < \epsilon.$$

Similarly, since $\lim_{n\to\infty} b_n = a$, there exists $N_2 > 0$ such that for all $n > N_2$,

$$|b_n - a| < \epsilon.$$

Let $N = \max\{N_1, N_2\}$ and suppose $m > N$.
If $m = 2k-1$ for some $k \in \mathbb{N}$, then $c_m = a_m$ and $m > N \geq N_1$, so that

$$|c_m - a| = |a_m - a| < \epsilon.$$

If $m = 2k$ for some $k \in \mathbb{N}$, then $c_m = b_m$ and $m > N \geq N_2$, so that

$$|c_m - a| = |b_m - a| < \epsilon.$$

It follows that for all $n > N$,

$$|c_m - a| < \epsilon,$$

so that $\lim_{m\to\infty} c_m = a$.

3. $a_{2m-1} = \left(1 - \frac{1}{2m-1}\right) \sin^2\left(m\pi - \frac{\pi}{2}\right) = \left(1 - \frac{1}{2m-1}\right)$ and

$$a_{2m} = \left(1 - \frac{1}{2m}\right) \sin^2(m\pi) = 0.$$

(a) Since $a_2 = 0 < a_3 = \dfrac{2}{3} > a_4 = 0$, it follows that (a_n) is not monotonic.

(b) Since $0 \leq a_{2m-1} = \left(1 - \dfrac{1}{2m-1}\right) < 1$ and $a_{2m} = 0$, it follows that $0 \leq a_n < 1$ for all $n \in \mathbb{N}$, so that (a_n) is bounded.

(c) $\text{glb}\{a_n : n \in \mathbb{N}\} = 0 \in \{a_n : n \in \mathbb{N}\}$ and

$$\text{lub}\{a_n : n \in \mathbb{N}\} = 1 \notin \{a_n : n \in \mathbb{N}\},$$

since $\lim_{m\to\infty} \left(1 - \dfrac{1}{2m-1}\right) = 1$.