1. Prove from first principals that $\{\frac{n+5}{3n+9}\}$ converges to $\frac{1}{3}$.

2. Let $(a_n)_{n=1}^{\infty}$ and $(b_n)_{n=1}^{\infty}$ be sequences. Let $(c_m)_{m=1}^{\infty}$ be the sequence such that

$$c_m = \begin{cases} a_m & \text{if } m = 2k-1, \\ b_m & \text{if } m = 2k. \end{cases}$$

Suppose that

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n = a$$

for some real number $a \in \mathbb{R}$. Prove from the first principles that $\lim_{m\to\infty} c_m = a$.

3. (**Final Exam of 2001SS**)

Let $(a_n)_{n=1}^{\infty}$ be the sequence such that $a_n = \left(1 - \dfrac{1}{n}\right)\sin^2\left(\dfrac{n\pi}{2}\right)$.

(a) Determine whether or not (a_n) is monotonic.

(b) Determine whether or not (a_n) is bounded.

(c) Find the greatest lower bound and the least upper bound of the set $\{a_n : n \in \mathbb{N}\}$ and determine whether or not either is an element of $\{a_n : n \in \mathbb{N}\}$.