1. If $f(x) = 0$ or $g(x) = 0$, then $f(x)g(x) = 0$ and

$$\deg(f(x)g(x)) = -\infty = \deg(f(x)) + \deg(g(x)).$$

(Note $-\infty + c = -\infty$ for any integer c or $c = -\infty$.)

Let $f(x) = a_0 + a_1x + \cdots + a_nx^n$ and $g(x) = b_0 + b_1x + \cdots + b_mx^m$ such that $\deg(f(x)) = n \neq -\infty$ and $\deg(g(x)) = m \neq -\infty$, so that $a_n \neq 0$ and $b_m \neq 0$. Then

$$f(x)g(x) = c_0 + c_1x + \cdots + c_{n+m}x^{n+m},$$

where $c_{n+m} = a_nb_m$. Since $a_n \neq 0$ and $b_m \neq 0$, it follows that $c_{n+m} \neq 0$, and so $\deg(f(x)g(x)) = m + n = \deg(f(x)) + \deg(g(x))$.

2. (a) Note the in $\mathbb{Z}_7$,

x	1	2	3	4	5	6
$3 \cdot_7 x$	3	6	2	5	1	4

Thus $3^{-1} = 5$ in $\mathbb{Z}_7$, and $f(x) = (3x^2 + 2)(5x^3 + 6x + 3) + (3x + 5)$. So $q(x) = 5x^3 + 6x + 5$, $r(x) = 3x + 5$.

(b)

$$\begin{aligned} f(x) &= g(x)(5x^3 + 6x + 3) + (3x + 5) \\ g(x) &= (3x + 5)(x + 3) + 1 \\ x + 3 &= (x + 3)1 + 0. \end{aligned}$$

Thus 1 is a greatest common divisor of $f(x)$ and $g(x)$. Moreover,

$$\begin{aligned} (3x + 5) &= f(x) - g(x)(5x^3 + 6x + 3) \\ 1 &= g(x) - (3x + 5)(x + 3) \\ &= g(x) - [f(x) - g(x)(5x^3 + 6x + 3)](x + 3) \\ &= f(x)(-x - 3) + g(x)(1 + (5x^3 + 6x + 3)(x + 3)) \\ &= f(x)(6x + 4) + g(x)(5x^4 + x^3 + 6x^2 + 3). \end{aligned}$$

Thus $u(x) = (6x + 4)$ and $v(x) = 5x^4 + x^3 + 6x^2 + 3$.

3. Since $y * x = y = y * e$, it follows by Cancellation Theorem that $x = e$. Thus $y * x = y * e = y$ for all $y \in G$.