1. Let $K \in \{\mathbb{Q}, \mathbb{R}, \mathbb{C}, \mathbb{Z}_p\}$ with p a prime. Let $f(x)$ and $g(x)$ be polynomials of $K[x]$. Show that

$$\deg(f(x)g(x)) = \deg(f(x)) + \deg(g(x)).$$

2. Let $f(x), g(x)$ and $h(x)$ be polynomials in $\mathbb{Z}_7[x]$ defined by

$$f(x) = x^5 + 2x^2 + x + 4, \qquad g(x) = 3x^2 + 2.$$

Here for simple, we denote $\bar{a}$ by a for $\bar{a} \in \mathbb{Z}_7$.

(a) Find quotient $q(x)$ and remainder $r(x)$ when $f(x)$ is divided by $g(x)$.

(b) Find a greatest common divisor $d(x)$ of $f(x)$ and $g(x)$,
and find polynomials $u(x)$ and $v(x)$ such that

$$d(x) = f(x)u(x) + g(x)v(x).$$

3. Let $(G, *)$ be a group with identity e and let $x \in G$. Show that if $y * x = y$ for some $y \in G$, then $y * x = y$ for all $y \in G$. [Hint: show that if $y * x = y$ for some $y \in G$, then $x = e$.]