1. Modified Euclidean Algorithm gives the following results:

n	x	y	
462	1	0	r_1
54	0	1	r_2
30	1	-1	$r_1 - 8r_2$
24	-1	9	$r_2 - r_3$
6	2	-17	$r_3 - r_4$
0	*	*	$r_4 - 4r_5$

Thus $\gcd(462, 54) = 6$ and $6 = 2 \cdot 465 + (-17) \cdot 54$.

2. (a) Let $c \in \mathbb{N}$ and suppose $c = p_1^{\gamma_1} p_2^{\gamma_2} \dots p_t^{\gamma_t}$, where each $\gamma_i \geq 0$. Then

$$\begin{aligned} ac = b &\iff p_1^{\alpha_1+\gamma_1} p_2^{\alpha_2+\gamma_2} \dots p_t^{\alpha_t+\gamma_t} = p_1^{\beta_1} p_2^{\beta_2} \dots p_t^{\beta_t} \\ &\iff \alpha_i + \gamma_i = \beta_i \qquad \text{by the uniqueness of FTA.} \end{aligned}$$

Thus $a \mid b \iff \alpha_i \leq \beta_i$.

(b) Let $d = p_1^{m_1} p_2^{m_2} \dots p_t^{m_t}$ and $D = \{x \in \mathbb{N} : x \mid a \ \wedge \ x \mid b\}$. Then $m_1 = \min\{\alpha_i, \beta_i\} \leq \alpha_i$, and by (a) above, $d \mid a$. Similarly, $d \mid b$ and $d \in D$.

Let $c = p_1^{\eta_1} p_2^{\eta_2} \dots p_t^{\eta_t} \in D$. Then $c \mid a$ and by (a) again, each $\eta_i \leq \alpha_i$. Similarly, each $\eta_i \leq \beta_i$, so that $\eta_i \leq m_i$ and $c \mid d$. In particular, $c \leq d$ and $d = \gcd(a, b)$.

Let $m = p_1^{M_1} p_2^{M_2} \dots p_t^{M_t}$ and $M = \{y \in \mathbb{N} : a \mid y \ \wedge \ b \mid y\}$. Then $M_i = \max\{\alpha_i, \beta_i\} \geq \alpha_i$, and by (a) above, $a \mid m$. Similarly, $b \mid m$ and $m \in M$.

Let $w = p_1^{\delta_1} p_2^{\delta_2} \dots p_t^{\delta_t} \in M$. Then $a \mid w$ and by (a) again, each $\alpha_i \leq \delta_i$. Similarly, each $\beta_i \leq delta_i$, so that $M_i = \max\{\alpha_i, \beta_i\} \leq \delta_i$ and $m \mid w$. In particular, $m \leq w$ and $m = \operatorname{lcm}(a, b)$.

Note each $m_i + M_i = \alpha_i + \beta_i$ and so

$$ab = p_1^{\alpha_1+\beta_1} p_2^{\alpha_2+\beta_2} \dots p_t^{\alpha_t+\beta_t} = p_1^{m_1+M_1} p_2^{m_2+M_2} \dots p_t^{m_t+M_t} = \gcd(a, b)\operatorname{lcm}(a, b).$$