**1.** Use the Euclidean algorithm to find the greatest common divisor of 462 and 54, and find integers $x$ and $y$ such that

$$\gcd(462, 54) = 462x + 56y.$$

**2.** Let $a, b \in \mathbb{N}$ and suppose $a = p_1^{\alpha_1} p_2^{\alpha_2} \ldots p_t^{\alpha_t}$ and $b = p_1^{\beta_1} p_2^{\beta_2} \ldots p_t^{\beta_t}$, where $p_1, \ldots, p_t$ are distinct primes and $\alpha_i, \beta_i$ are nonnegative integers.

(a) Show that $a \mid b$ if and only if $\alpha_i \leq \beta_i$ for all $i$.

(b) Let $m_1 = \min\{\alpha_i, \beta_i\}$ and $M_i = \max\{\alpha_i, \beta_i\}$. Show that

$$\gcd(a, b) = p_1^{m_1} p_2^{m_2} \ldots p_t^{m_t} \qquad \operatorname{lcm}(a, b) = p_1^{M_1} p_2^{M_2} \ldots p_t^{M_t}.$$

Check $\gcd(a, b)\operatorname{lcm}(a, b) = ab$.