1. Indicate whether each of the following relations on the given set is reflexive, symmetric, anti-symmetric, or transitive. Explain your answers.

(a) $A = \mathcal{P}(\{1, 2, \cdots, 10\})$ and let the relation $\sim$ on A be defined as follows: for all $X, Y \in A$, $X \sim Y$ iff for all $x \in X$, there exists $y \in Y$ such that $x|y$.

(b) $B = \{x \in \mathbb{Z} : x \geq 0\}$ and $x\rho y$ iff $x + y = 0$.

2. Let S be the set $\{a, b, c, d, e, f\}$.

(a) Let ρ be the relation

$$\rho = \{(a, a), (b, b), (c, c), (d, d), (e, e), (f, f), (a, b), (b, a), (a, c), (c, a), (b, c), (c, b), (d, e), (e, d)\}$$

Verify that ρ is an equivalence relation. Find all equivalence classes and check the collection of distinct classes is a partition of S.

(b) Let $S_1 = \{a\}$, $S_2 = \{b, d, f\}$ and $S_3 = \{c, e\}$. Verify that $\{S_1, S_2, S_3\}$ is a partition of S. Define an equivalence relation ρ such that each S_i is an ρ-equivalence class.

3. Let $A = \mathbb{Z}$ and $\sim$ be a relation on A such that $x \sim y$ iff $x + 3y$ is even.

(a) Show $\sim$ is an equivalence relation.

(b) Show that $[0], [1]$ are all distinct equivalence classes.