1.

$$\begin{aligned} x \in (A \cup B)^C_U &\iff x \in U \wedge (x \notin A \cup B) \\ &\iff x \in U \wedge \sim(x \in A \wedge x \in B) \\ &\iff (x \in U) \wedge ((x \notin A) \vee (x \notin B)) \\ &\iff (x \in U \wedge x \notin A) \wedge (x \in U \wedge x \notin B) \\ &\iff x \in A^C_U \cap B^C_U. \end{aligned}$$

Thus

$$(A \cup B)^C_U = A^C_U \cap B^C_U.$$

2.

$$\begin{aligned} x \in (A \setminus B) &\iff x \in A \wedge (x \notin B) \\ &\iff x \in A \wedge (x \in U \wedge x \notin B) \\ &\iff x \in A \wedge (x \in B^C_U) \\ &\iff x \in A \cap B^C_U. \end{aligned}$$

Thus

$$A \setminus B = A \cap B^C_U.$$

3.

$$\begin{aligned} x \in A \setminus \bigcup_{\alpha \in \Lambda} B_\alpha &\iff x \in A \wedge x \notin \bigcup_{\alpha \in \Lambda} B_\alpha \\ &\iff x \in A \wedge \sim(x \in \bigcup_{\alpha \in \Lambda} B_\alpha) \\ &\iff x \in A \wedge \sim(\exists \alpha \in \Lambda)(x \in B_\alpha) \\ &\iff x \in A \wedge (\forall \alpha \in \Lambda)(x \notin B_\alpha) \\ &\iff (\forall \alpha \in \Lambda)(x \in A \wedge x \notin B_\alpha) \\ &\iff (\forall \alpha \in \Lambda)(x \in A \setminus B_\alpha) \\ &\iff x \in \bigcap_{\alpha \in \Lambda} (A \setminus B_\alpha) \end{aligned}$$

4. $\mathcal{P}(\{1\}) = \{\emptyset, \{1\}\}$ and

$$\mathcal{P}(\mathcal{P}(\{1\})) = \{\emptyset, \{\emptyset\}, \{\{1\}\}, \{\emptyset, \{1\}\}\}.$$