1. (a) (**6 marks**)

$*$	a	b	c	d
a	c	d	a	b
b	d	c	b	a
c	a	b	c	d
d	b	a	d	c

(b) (**4 marks**) $e_G = c$, $b^{-1} = b$ and $d^{-1} = d$ in G.

2. (**8 marks**) Since $\det I_2 = 1$, it follows that $I_2 \in H$ and $H \neq \emptyset$.

For $A, B \in H$, $\det(A) = \det(B) = 1$ and so

$$\det(AB^{-1}) = \det(A)\det(B^{-1}) = \det(A)\det(B)^{-1} = 1.$$

Thus $AB^{-1} \in H$ and $H \leq \mathrm{GL}_2(\mathbb{R})$ by One-step subgroup test.

3. (a) (**4 marks**)

$+_4$	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

(b) (**6 marks**) Construct a Cayley table of G is given as follows:

$\cdot$	1	i	-1	$-i$
1	1	i	-1	$-i$
i	i	-1	$-i$	1
-1	-1	$-i$	1	i
$-i$	$-i$	1	i	-1

Define $\psi(0) = 1, \psi(1) = i, \psi(2) = -1, \psi(3) = -i$. Then ψ is a bijection from $\mathbb{Z}_4$ onto G, and moreover, ψ sends the Cayley table of $\mathbb{Z}_4$ to the Cayley table of G, so $\psi(x +_4 y) = \psi(x)\psi(y)$ and ψ is an isomorphism.

4. (a) (**4 marks**) Since $\alpha^{-1} = \alpha$ and $e^{-1} = e$ in S_3, it follows by the following Cayley table of K that $ab^{-1} \in K$ for any $a, b \in K$.

$\cdot$	e	α
e	e	α
α	α	e

Thus $K \leq S_3$.

(b) (**8 marks**) $eK = K = \alpha K$, $\varphi K = \{\varphi, \beta\} = \beta K, \psi K = \{\psi, \gamma\} = \gamma K$.