MATHS 255 | Assignment 4 Solutions | Due: 19 August, 2003

1. [12 marks] We first show $<$ is antireflexive: Suppose x in S and $x < x$. Then $x \le x$ and $x \ne x$. This is a contradiction, so $x < x$ is false.

To show that $<$ is transitive: Suppose that x,y,z are in S and $x < y$ and $y < z$. Then $x \le y$ and $y \le z$, and $x \ne y$ and $y \ne z$. Hence $x \le z$ (since $\le$ is transitive) and if $x = z$, then we have both $y \le z$ and $x = z \le y$, so that $y = z$ (since $\le$ is antisymmetric). This contradicts that $y < z$, so it is false that $x = z$. We have therefore shown that $x < z$.

Hence, $<$ is a quasi-order relation.

2, [7 marks]

a) Maximal elements of A: *h*. Maximal elements of B: *o,p,q,r*. Maximal elements of C: *z*.

(b) Minimal elements of A: *a,b,c*. Minimal elements of B: *j*. Minimal elements of C: *u*.

(c) Smallest element of *A*: none. Smallest element of *B*: *j*. Smallest element of *C*: *u*.

(d) *f,g*.

(e) Upper bounds: none. Lower bounds: *j, k*.

(f) lub{*d,c*}: *f*.. lub{*w,y.v*}: *z*. lub{*p,k*}: *p*. glb{*a,g*}: none. glb{*p,n*}: *k*.

(g) 18 pairs altogether: {*(u,u) (u,v) (u,w) (u,x) (u,y) (u,z) (v,v) (v,w) (v,x) (v,y) (v,z) (w,w) (w,z) (x,x) (x,z) (y,y) (y,z) (z,z)*}

3. [12 marks] First we show that g is an upper bound of A. Suppose not. Then for some $a \in A$, $g < a$. But for all $x \in U$, x is an upper bound of A, hence $a \le x$. Hence a is a lower bound for U, and hence $a \le g$ (since $g =$glb U).

This contradicts $g < a$. So g is an upper bound for A.

Now if h is any upper bound of A, then $g \le h$ (g is a lower bound for U and h is in U). Hence g is a least upper bound of A.

4. (a) [4 marks] $|z - i| = |w - i|$ means that the distance from z to i is the same as the distance from w to i. The distance from $4+4i$ to i is 5. So the equivalence class containing $4+4i$ is all points whose distance from i is 5, namely a circle centred at i of radius 5.

(b) [5 marks] $A\begin{pmatrix} x \\ y \end{pmatrix} = A\begin{pmatrix} w \\ z \end{pmatrix}$ if and only if $x - 2y = w - 2z$. So given a point (a,b), The points related to it by this equivalence relation are the points on the line with equation $x - 2y = a - 2b$, or in another form $y = \frac{x}{2} - \frac{a-2b}{2}$, that is the line with gradient 1/2 through the point (a,b). The nullspace of A is all vectors of the form $\begin{pmatrix} x \\ y \end{pmatrix} = t\begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $t \in \mathbf{R}$ which is a parametric equation of the line with slope 1/2 through the origin, i.e. the equivalence class containing (0,0).