1. (**5 marks**) Suppose D_5 has a subgroup of order 5. Then by Lagrange's theorem, 5 divides $|D_4| = 8$, which is impossible.

2. (a) (**3 marks**) Suppose $ab = 0$ and $a \neq 0$. Then there is an element $c \in F$ such that $ca = 1_F$. Thus $b = 1_F b = (ca)b = c(ab) = c0_F = 0_F$.

(b) (**3 marks**) Note $-a = (-1_F)a$, since $a+(-a) = 0_F = 0_F a = (1_F+(-1_F))a = a+(-1_F)a$. Thus $-ab = ((-1_F)a)b = (-1_F)(ab) = -(ab)$, and $(-a)(-b) = ((-1_F)a)((-1_F)b) = -(-1_F)ab = ab$.

(c) (**3 marks**) $a/b+c/d = ab^{-1}+cd^{-1} = ab^{-1}dd^{-1}+cd^{-1}bb^{-1} = (ad+cb)b^{-1}d^{-1} = (ad+cb)(bd)^{-1} = (ad + cb)/bd$.

(d) (**2 marks**) $(a/b)(c/d) = ab^{-1}cd^{-1} = acb^{-1}d^{-1} = ac(bd)^{-1} = ac/bd$.

(e) (**2 marks**) $(a/b)/(c/d) = ab^{-1}(cd^{-1})^{-1} = ab^{-1}c^{-1}d = ad(bc)^{-1} = ad/bc$.

3. (a) (i) (**3 marks**) Suppose $a < b$. Then $b - a \in P$ and $-c = 0 - c \in P$, so that $-cb + ca = (-c)(b - a) \in P$ and $cb < ca$.

(ii) (**3 marks**) Since $-1 < 0$, it follows by (i) above that if $a < b$, then $(-1)b < (-1)a \iff -b < -a$. Thus if $a < 0$, then $-a = (-1)a > 0$, as $-1 < 0$.

(iii) (**4 marks**) Suppose $ab > 0$. If $a > 0$ and $b < 0$, then by (i), $ab < 0 \cdot b = 0$.
If $a > 0$ and $b > 0$, then $ab > 0$.
If $a < 0$ and $b > 0$, then by (i), $ab < 0 \cdot a = 0$.
If $a < 0$ and $b < 0$, then by (i), $ab > 0 \cdot b = 0$.

(b) (**4 marks**) Suppose $\mathbb{C}$ is ordered. Then there is a subset P such that $1 \in P$. But $x^2 \in P$ for any $x \in \mathbb{C}$ with $x \neq 0$, so $-1 = i^2 \in P$ and $0 = 1 + (-1) \in P$, which is impossible.

4. (**8 marks**) Let U_S be the set of all upper bounds of S. Then $T \subseteq U_S$ and in particular, S is bounded above. Thus lub S exists, and is the least element of U_S. Since $T \subseteq U_S$, lub $S \leq y$ for each $y \in T$.

Let L_T be the set of lower bounds of T. As shown above, lub $S \in L_T$ and in particular, T is bounded below. Thus glb T exists, and is the greatest element of L_T. Since lub $S \in L_T$, lub $S \leq$ glb T.